Sum of Arithmetic Series

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Ex: 2, 5, 8, ..., 11 = 36
2 + 5 + 8 + ... + 11

Deriving the Formula

$t_n = a_1 + (n-1)d$

t_n = term
a = First term
d = common difference
n = term #

$S_n = a_1 + a_2 + a_3 \cdots a_n$

Let $l = a + (n-1)d$ (last term)

$S_n = a_1 + (a_1+d) + (a_1+2d) + \ldots + (a_1+(n-1)d)$

(Eq1) $S_n = a_1 + (a_1+d) + (a_1+2d) + \cdots + l$

(Eq2) $S_n = l + (l-d) + (l-2d) + \ldots + a_1$

$Eq1 + Eq2 \Rightarrow$ $S_n + S_n = a_1 + a_1 + d + a_1 + 2d + \cdots + l + l + (l-d) + (l-2d) + \cdots + a_1$

$2S_n = (a_1 + l)n$

$\frac{2S_n}{2} = \frac{(a_1+l)n}{2}$

$S_n = \frac{n}{2}(a_1 + l)$

$$S_n = \frac{n}{2}(a+l) = \frac{n}{2}(2a + (n-1)d)$$

a = First term
l = last term
d = common difference
n = # of terms

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* Example: Find the Sum

(1) $a = 1$, $d = 1$, $l = 50$, $n = 50$

$S_n = \frac{n}{2}(a+l)$

$= \frac{50}{2}(1+50) = 25(51)$

$= 1275$

* (2) Find the Sum for the first 25 numbers in this Series

$11 + 15 + 19 + \ldots$

$a = 11$, $n = 25$, $d = 4$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{25}{2}(2(11) + (25-1)4)$

$= \frac{25}{2}(118) = 1475$

* (3) Find the Sum of this Series $7 + 10 + 13 + \ldots + 100$

$a = 7$, $d = 3$

$t_n = a + (n-1)d$

$100 = 7 + (n-1)3$

$S_n = \frac{n}{2}(a+l)$

$a = 7$
$d = 3$
$l = 100$

$t_n = a + (n-1)d$
$100 = 7 + (n-1)3$
$-7 \quad -7$
$\frac{93}{3} = \frac{(n-1)(3)}{3}$
$31 = n-1$
$32 = n$

$S_n = \frac{n}{2}(a+l)$
$= \frac{32}{2}(7+100)$
$= 16(107)$
$= 1712$

* ④ Evaluate

$$\sum_{k=1}^{100} (2k+1)$$

$a_1 = 2(1)+1 = 3$
$l = a_{100} = 2(100)+1 = 201$
$n = 100$

$S_n = \frac{n}{2}(a+l)$
$= \frac{100}{2}(3+201)$
$= 50(204)$
$= 10200$

* ⑤ write in Summation notation (Σ)

$5 + 9 + 13 + \ldots + 137$
$a \qquad\qquad\quad l$

$l = a + (n-1)d$
$137 = 5 + (n-1)4$
$-5 \quad -5$
$\frac{132}{4} = \frac{(n-1)4}{4}$
$33 = n-1$
$n = 34$

$t_n = a + (n-1)d$
$t_n = 5 + (n-1)4$

$$\sum_{n=1}^{34} t_n$$
$$\sum_{n=1}^{34} [5 + (n-1)4]$$
$$\sum_{n=1}^{34} [4n+1]$$

* ⑥ Find the common difference of an arithmetic Sequence with a sum of $S_n = 5n^2 - 3n$.

$S_1 = a_1 = 5(1)^2 - 3(1)$
$a_1 = 5 - 3$
$a_1 = 2$

$S_2 = a_1 + a_2 = 5(2)^2 - 3(2)$
$= 5(4) - 6$
Sub. → $a_1 + a_2 = 14$
$2 + a_2 = 14$
$a_2 = 12$

$d = a_2 - a_1$
$d = 12 - 2$
$d = 10$

* ⑦ Find the Sum of all multiples of 6 between 100 and 1000

$a =$ First multiple $= 102$
$l =$ Last multiple $= 996$
$n =$

$t_n = a + (n-1)d$
$996 = 102 + (n-1)6$
$\frac{996-102}{6} = \frac{6(n-1)}{6}$
$149 = n-1$
$150 = n$

$$S_n = \frac{n}{2}(a+l)$$
$$= \frac{150}{2}(102+996)$$
$$= \underline{\underline{82\,350}}$$

$t_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(a+l)$ $\quad S_n = \frac{n}{2}(2a+(n-1)d)$

t_n: term
a: First term
l: Last term
n: term number
d: common difference
S_n: arithmetic Sum

work on 8.2 Exercises