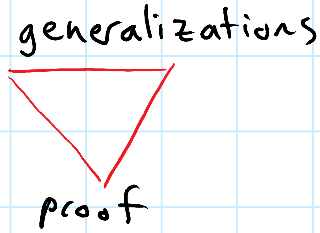
# Deductive Reasoning

September 12, 2016 7:29 PM




**Deductive Reasoning:** Drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid.

**Proof:** An argument that shows a statement to be true in all cases. (No counterexamples exists)

**Generalization:** A principle, statement, or idea that has general application.

**Using Deductive Reasoning**

In order to use deductive reasoning to prove a conjecture, we must first generalize all the evidence. Then we prove that ~~that~~ generalization is true for all cases.

In other words: we prove our conjecture works for all examples by proving it true for the generalization.

**Mathematical Examples**

**Ex1:**

Jon discovered a pattern when adding integers

$1 + 2 + 3 + 4 + 5 = 15$ $\quad 3 \times 5 = 15$

$(-15) + (-14) + (-13) + (-12) + (-11) = -65$ $\quad -13 \times 5 = -65$

$(-3) + (-2) + (-1) + 0 + 1 = -5$ $\quad -1 \times 5 = -5$

His conjecture is that whenever you add five consecutive integers, the sum is always 5 times the middle number.

generalize the problem

let the middle number $= X$

$$(x-2) + (x-1) + X + (x+1) + (x+2) = \text{Sum}$$

$$x + x + x + x + x = \text{Sum}$$

$$5x = \text{Sum}$$

$\therefore$ The Sum of 5 consecutive numbers is just 5 times the middle number

**Ex2:**

Prove that an odd number + another odd number is always even.

let $x$ be any integer

$2x$: Even #

Prove that an odd number + another odd number is always even.

$2x$: Even #
$2x+1$: odd #

Let $x$ be any integer
then $(2x+1)$ will be odd
Let $y$ be any integer
then $(2y+1)$ will be odd

$$(2x+1)+(2y+1) = \underbrace{2x}_{\text{Even}} + \underbrace{2y}_{\text{Even}} + \underbrace{2}_{\text{Even}}$$

$\therefore$ An odd + another odd will be even

**Ex3:**
Prove that the difference between consecutive squares is always an odd number.

Let $x$ be any integer
then consecutive squares are $x^2$ and $(x+1)^2$

$$D = (x+1)^2 - x^2$$
$$= (x+1)(x+1) - x^2$$
$$= \cancel{x^2} + x + x + 1 - \cancel{x^2}$$
$$= 2x+1$$

First
Outer
Inner
Last

$\therefore$ we will always get an odd number from the difference of consecutive squares

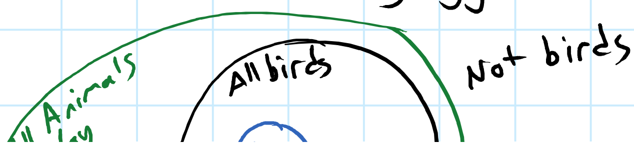
<u>**Using Deductive Reasoning outside of math**</u>
To use Deductive Reasoning outside of math equations, we need to introduce the term Premise.

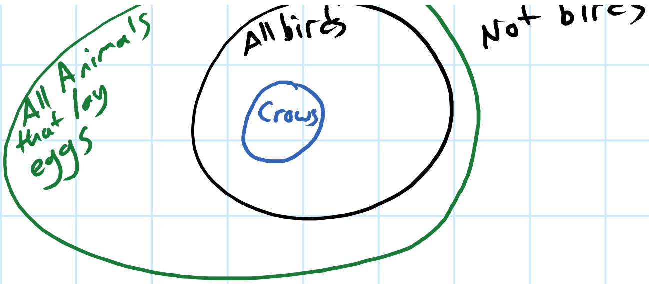
**Premise:** A statement that is assumed to be true for all cases. (i.e. All crows are birds or all birds lay eggs etc.)

We can use premises to link information and prove a specific fact about something.

**Ex:** If all crows are birds and all birds lay eggs then what can we conclude about crows?

$\therefore$ All crows lay eggs






**Advance Ex**: If the sum of the digits of a number is divisible by three then so too is the original number.
Prove this for the two digit case

$$93 = 9 \times 10 + 3$$

Let $x, y$ be the digits of the number ie. $xy$

$$\text{The number} = 10x + y$$
$$= (9x + x) + y$$
$$= 9x + x + y$$
$$\text{The number} = 9x + (x + y)$$

$9x$ is divisible by 3
So when $x+y$ is divisible by 3 our original number $xy$ will also be divisible by 3.

Pg. 31 : odd