

# Geometric Sequences

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Each new term in a geometric sequence is found by multiplying the previous term by a number. This number is called the common ratio (r).

Ex: 3, 6, 12, 24, 48, ...

Common Ratio = 2

## Geometric Sequence

A sequence is geometric if the ratio of the consecutive terms is constant.

ie.  $r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \dots = \frac{a_n}{a_{n-1}} ; r \neq 0$

General Formula for any term

$$t_n = ar^{(n-1)}$$

$t_n$ :  $n^{\text{th}}$  term  
 $a$ : First term  
 $n$ : term number

Ex: Find the Common Ratio

a) 4, 12, 36, ...

$$r = \frac{12}{4} = 3$$

$$r = \frac{36}{12} = 3$$

$$r = 3$$

b) -4, 2, -1, ...

$$r = \frac{2}{-4} = -\frac{1}{2}$$

Ex: Find the 7<sup>th</sup> term in the following Sequence

③ -6, 12, -24, ...

$$a = 3$$

$$r = \frac{-6}{3} = -2$$

$$t_n = ar^{n-1}$$

$$t_7 = 3(-2)^{6}$$

$$= 3(64)$$

$$t_7 = 192$$

Ex: The 3<sup>rd</sup> term in a geometric sequence is -2 and the 6<sup>th</sup> term is  $-\frac{1}{4}$ . Find the 10<sup>th</sup> term.

②  $t_3 = ar^{(3-1)}$   
 $-2 = ar^2$

$t_6 = ar^{(6-1)}$   
 $-\frac{1}{4} = ar^5$

Sub  $\frac{t_6}{t_3}$

$$\frac{-\frac{1}{4}}{-2} = \frac{ar^5}{ar^2}$$

$$\frac{1}{8} = r^3$$

$$\sqrt[3]{\frac{1}{8}} = \sqrt[3]{r^3}$$

$$\frac{1}{2} = r$$

Sub  $\frac{t_6}{t_3}$

$$\frac{-\frac{1}{4}}{-2} = \frac{a(\frac{1}{2})^5}{a(\frac{1}{2})^2}$$

$$-\frac{1}{8} = \frac{a(\frac{1}{2})^3}{a(\frac{1}{2})^2}$$

$$-\frac{1}{8} = a(\frac{1}{2})$$

$$-8 = a$$

$$t_n = ar^{(n-1)}$$

$$t_{10} = (-8)(\frac{1}{2})^9$$

$$= (-8)(\frac{1}{2})^9$$

$$= (-8)(\frac{1}{512})$$

$$= -\frac{1}{64}$$

\* Factoring

Ex: What value of x in the following will form a geometric Sequence.

Ex: What value of  $x$  in the following will form a geometric Sequence.

$$\begin{matrix} a_1 & a_2 & a_3 \\ x, & 2x+2, & 3x+3 \end{matrix}$$

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} \quad \rightarrow \quad \left( \frac{2x+2}{x} \right) = \left( \frac{3x+3}{2x+2} \right) x$$

$$(2x+2)(2x+2) = \left[ \frac{(3x+3)(x)}{2x+2} \right] (2x+2)$$

$$(2x+2)(2x+2) = (3x+3)(x)$$

$$4x^2 + 4x + 4x + 4 = 3x^2 + 3x$$

$$4x^2 + 4x + 4x + 4 - 3x^2 - 3x = 0$$

$$x^2 + 5x + 4 = 0$$

$$(x+1)(x+4) = 0$$

$$x+1=0$$

$$-1 -1$$

$$x = -1$$

$$x+4=0$$

$$-4 -4$$

$$x = -4$$

First  
Outer  
Inner  
Last

$x$	$+$
4	5
$1 \times 4$	5