Sum of Geometric Sequences

September 13, 2016 8:12 PM

$\boxed{8.4}$

$$S_n = \sum_{k=1}^{n} ar^{k-1}$$

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

Deriving the formula

$$-rS_n = -ar - ar^2 - ar^3 - \dots - ar^{n-1} - ar^n$$

$$S_n - rS_n = a + \cancel{ar} + \cancel{ar^2} + \dots + \cancel{ar^{n-1}} - \cancel{ar} - \cancel{ar^2} - \dots - \cancel{ar^{n-1}} - ar^n$$

$$S_n - rS_n = a - ar^n$$

$$\frac{S_n(1-r)}{1-r} = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Formula: Geometric Sum

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

a: first term
n: term number
r: common ration

$\boxed{\text{or}}$

$$S_n = \frac{a - rl}{1-r}$$

l: last term

$\boxed{1d}$ $S_6 =$

$a = -8$

$t_4 = 27$

$$t_n = ar^{n-1}$$

$$27 = (-8)r^{4-1}$$

$$\sqrt[3]{27} = \sqrt[3]{(-8)r^3}$$

$$\frac{3}{-2} = \frac{-2r}{-2}$$

$$-\frac{3}{2} = r$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_6 = \frac{-8\left(1-\left(\frac{-3}{2}\right)^6\right)}{1-\left(\frac{-3}{2}\right)}$$

$$= \frac{-8\left(1-\frac{729}{64}\right)}{1+\frac{3}{2}}$$

$$= 33.25$$

$\boxed{2d}$ h

$= 33.25$

2d $\sum_{i=a}^{b} (2^i - 5)$ $\qquad \underline{b-a+1}$

3f $\sum_{i=3}^{11} \frac{2^i}{3^{i-1}}$

$n = 11-3+1$

$\underline{n = 9}$

$a_1 = \frac{2^3}{3^{3-1}} = \frac{2^3}{3^2}$ // i=3

$a_2 = \frac{2^4}{3^{4-1}} = \frac{2^4}{3^3}$ // i=4

$r = \frac{a_2}{a_1} = \frac{\frac{2^4}{3^3}}{\frac{2^3}{3^2}}$

$= \left(\frac{2^4}{3^3}\right)\left(\frac{3^2}{2^3}\right)$

$\boxed{r = \frac{2}{3}}$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_n = \left(\frac{2^3}{3^2}\right)\frac{\left[1-\left(\frac{2}{3}\right)^9\right]}{\left[1-\frac{2}{3}\right]}$

$= \left(\frac{8}{9}\right)\left(\frac{1-\frac{512}{19683}}{1-\frac{2}{3}}\right)$

$= \left(\frac{8}{9}\right)\left(\frac{\frac{19683-512}{19683}}{\frac{1}{3}}\right)$

$= \left(\frac{8}{9}\right)\left(\frac{19171}{19683}\right)\left(\frac{3}{1}\right)$

$= \underline{2.597}$

4c $\frac{3}{16} - \frac{3}{8} + \frac{3}{4} - \ldots - 384$

$r = \frac{-\frac{3}{8}}{\frac{3}{16}} = \left(\frac{-3}{8}\right)\left(\frac{16}{3}\right)$

$= -\frac{16}{8}$

$\underline{r = -2}$

$t_n = ar^{n-1}$

$\left[-384 = \left(\frac{3}{16}\right)(-2)^{n-1}\right] \times \left(\frac{16}{3}\right)$

$-2048 = (-2)^{n-1}$

$(-2)^{11} = (-2)^{n-1}$

$11 = n-1$

$+1 \qquad +1$

$\underline{12 = n}$

$\sum^{12} t_k = \sum^{12} \left(\frac{3}{\ }\right) r^{k-1}$

$$\sum_{k=1}^{12} t_k = \sum_{k=1}^{12} \left(\frac{3}{16}\right) r^{k-1}$$

11) a) $40000(1.05)^5 = \$51{,}051$ ← Higher

$43000(1.03)^5 = \$49{,}849$

b) $\sum_{k=1}^{5} 40000(1.05)^{k-1}$ $\quad S_5 = 40000\dfrac{(1-1.05^5)}{1-1.05}$

$= \underline{\underline{\$221{,}025}}$

$\sum_{k=1}^{5} 43000(1.03)^{k-1}$ $\quad S_5 = 43000\dfrac{(1-1.03^5)}{1-1.03}$

$= \underline{\underline{\$228{,}293}}$ ← Higher