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CHAPTER 8 SEQUENCES AND SERIES PRE-TEST

NAME: KEY

Pre-Calculus 11

$$t_n = t_1 + (n-1)d, \quad S_n = \frac{(t_1 + t_n)}{2}n, \quad t_n = t_1 r^{n-1}, \quad S_n = \frac{t_1(1-r^n)}{1-r}, \quad S_n = \frac{t_1}{1-r}$$

1. Find the term t_{23} from the sequence: $-23, -17, -11, \dots$

$$\begin{aligned} d &= -17 - (-23) = 6 \\ t_1 &= -23 \\ t_{23} &= ? \end{aligned} \quad \left| \quad \begin{aligned} t_{23} &= -23 + (23-1)6 \\ t_{23} &= 109 \end{aligned} \right.$$

Answer: (2 marks)

$$t_{23} = 109$$

2. Find the general formula from the sequence: $5, -3.5, -12, \dots$

$$\begin{aligned} d &= -3.5 - 5 = -8.5 \\ t_1 &= 5 \\ t_n &= ? \end{aligned} \quad \left| \quad \begin{aligned} t_n &= 5 + (n-1)(-8.5) \end{aligned} \right.$$

Answer: (2 marks)

$$t_n = 5 + (n-1)(-8.5)$$

3. For the arithmetic sequence: $23, 25.6, 28.2, \dots, 340.2$, which term is 340.2?

$$\begin{aligned} d &= 25.6 - 23 = 2.6 \\ t_1 &= 23 \\ t_n &= 340.2 \\ n &= ? \end{aligned} \quad \left| \quad \begin{aligned} t_n &= t_1 + (n-1)d \\ 340.2 &= 23 + (n-1)2.6 \\ n &= \frac{340.2 - 23}{2.6} + 1 = 123 \end{aligned} \right.$$

Answer: (2 marks)

$$n = 123$$

4. In the sequence the 5th term is 7 and the 14th term is -20 . Find the first 4 terms.

$$\begin{array}{cccccccccccccccc} & +3 & +3 & +3 & +3 & & & & & & & & & & & & \\ 19 & 16 & 13 & 10 & 7 & & & & & & & & & & & & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$

$$-20 - 7 = 9d$$

$$d = \frac{-27}{9} = -3$$

Answer: (2 marks)

$$19, 16, 13, 10$$

5. For the arithmetic sequence: $-70, -55, \overset{-40}{-40}, \dots$, how many terms are less than 2048?

$$\begin{aligned} d &= -55 - (-70) = 15 & 2048 > -70 + (n-1)(15) \\ t_1 &= -70 & n < \frac{2048 + 70}{15} + 1 \\ t_n &< 2048 & n < 142.2 \\ n &=? \end{aligned}$$

Answer: (2 marks)

$$n = 142$$

6. Find the sum of the first 21 terms of the series: $-140 + -121 + -102, \dots$

$$\begin{aligned} d &= -121 - (-140) = 19 & t_{21} &= -140 + (21-1)(19) \\ t_1 &= -140 & t_{21} &= 240 \\ n &= 21 & S_{21} &= \left(\frac{-140 + 240}{2} \right) (21) = \\ S_n &=? \end{aligned}$$

Answer: (2 marks)

$$S_{21} = 1050$$

7. Find the sum for the series: $14 + 8 + 2, \dots, -184$

$$\begin{aligned} t_1 &= 14 & -184 &= 14 + (n-1)(-6) \\ t_n &= -184 & n &= 34 \\ d &= 8 - 14 = -6 & S_n &= \left(\frac{14 + (-184)}{2} \right) 34 = \\ n &=? \end{aligned}$$

Answer: (2 marks)

$$S_n = -2890$$

8. A rubber ball is dropped from a very high tower and it takes 12 seconds to hit the ground. The ball falls 2.4 m in the first second, 10 m in the second, 17.6 m in the third second and so on. How tall is this tower?

$$\begin{aligned} d &= 10 - 2.4 = 7.6 \text{ m} & t_n &= 2.4 + (12-1)7.6 \\ t_1 &= 2.4 \text{ m} & t_n &= 86 \\ t_n &=? & S_{12} &= \left(\frac{2.4 + 86}{2} \right) 12 \\ n &= 12 & S_{12} &= 530.4 \text{ m} \end{aligned}$$

Answer: (2 marks)

$$S_{12} = 530.4 \text{ m}$$

9. What is t_7 of the sequence: 62, -31, 15.5, ...

$$r = -31 \div 62 = -\frac{1}{2} \quad t_7 = 62 \left(-\frac{1}{2}\right)^{7-1}$$

$$t_1 = 62$$

$$t_7 = ?$$

$$n = 7$$

$$t_7 = 0.96875$$

Answer: (2 marks)

$$t_7 = 0.96875$$

10. If the value of Mr. Roome's \$8000 car depreciates by 8% per year, then how much would it be worth 9 years later?

$$r = 100\% - 8\% = 92\% = .92$$

$$n = 9 + 1 = 10 \text{ (10th term)}$$

$$t_1 = 8000$$

$$t_{10} = 8000 (.92)^{10-1}$$

$$t_{10} = 3777.29$$

Answer: (2 marks)

$$t_{10} = \$3,777.29$$

11. Determine the sum of the first 7 terms of the sequence: 3, -12, 48, ...

$$t_1 = 3$$

$$r = -12 \div 3 = -4$$

$$S_7 = ?$$

$$S_7 = \frac{3(1 - (-4)^7)}{1 - (-4)}$$

$$S_7 = 9831$$

Answer: (2 marks)

$$S_7 = 9,831$$

12. Determine the sum of $\sum_{i=4}^{14} \left(\frac{2}{3}\right)^i$

$$r = \frac{2}{3}$$

$$n = 14 - 4 + 1 = 11$$

$$t_1 = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

$$S_{11} = \frac{\frac{16}{81} \left(1 - \left(\frac{2}{3}\right)^{11}\right)}{1 - \frac{2}{3}} = 0.586$$

Answer: (2 marks)

$$S_{11} = 0.586$$

13. Find t_1 from the sequence that adds up to 215 with a common ratio of $\frac{2}{3}$

$$S_n = 215$$

$$r = \frac{2}{3}$$

$$S = \frac{t_1}{1 - r}$$

$$t_1 = S \cdot (1 - r) \\ = 215 \left(1 - \frac{2}{3}\right) = 71\frac{2}{3}$$

Answer: (2 marks)

$$t_1 = 71\frac{2}{3} \quad \text{OR} \quad (71.6667)$$

14. Use the formula for an **infinite** geometric series to find fraction for 0.42424242...

$$S = \frac{42}{100} + \frac{42}{10000} + \frac{42}{1000000} + \dots$$

$$t_1 = \frac{42}{100}$$

$$S = \frac{\frac{42}{100}}{1 - \frac{1}{100}} = \frac{\frac{42}{100}}{\frac{99}{100}}$$

$$r = \frac{1}{100}$$

$$S = \frac{42}{100} \div \frac{99}{100} \Rightarrow \frac{42}{100} \times \frac{100}{99} = \frac{42}{99}$$

End of Test

Answer: (2 marks)

$$\frac{42}{99}$$