Radical Operations

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nth Root

If a and b are real numbers and n is a positive integer, then x is an nth root of a if $x^n = a$.

$$x^n = a \rightarrow x = a^{\frac{1}{n}} \quad \text{or} \quad x = \sqrt[n]{a}$$

Exponent form   Radical form

Ex: $x^3 = 8$
$x = 8^{\frac{1}{3}}$
$x = 2$

If a is positive and n is even then we will have two real roots.

Ex: $x^2 = 9 \rightarrow x = \sqrt{9} = \pm 3$

$x^2 = 6 \rightarrow x = \pm\sqrt{6}$ or $\pm 6^{1/2}$

$x^4 = 3 \rightarrow x = \pm\sqrt[4]{3}$ or $\pm 3^{1/4}$

If a is negative and n is even then there are no real roots.

$x^2 = -5 \rightarrow$ No real Solution

If n is odd then there is always one real root.

$x^3 = -8 \rightarrow x = \sqrt[3]{-8} = -2$

$x^5 = 3125 \rightarrow x = \sqrt[5]{3125} = 5$

If a is zero then the root is always zero.

$x^5 = 0 \rightarrow x = 0$

Exponential Rules Review

Multiplying Exponential Terms: add the exponents

$$(x^5)(x^2) = x^{5+2} = x^7$$

$$(\sqrt{x})(x^2) = x^{\frac{1}{2}}x^2 = x^{\frac{1}{2}+\frac{4}{2}} = x^{5/2}$$

Dividing Exponential Terms: subtract the exponents

$$\frac{x^5}{\ } = x^{5-2} = x^3$$

Dividing Exponential Terms: subtract the exponents

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

$$\frac{\sqrt{x}}{x^2} = x^{\frac{1}{2}-2} = x^{\frac{1}{2}-\frac{4}{2}} = x^{-3/2} = \frac{1}{x^{3/2}}$$

Taking Exponents of Exponential Terms: multiply exponents

$$(\sqrt{x})^6 = (x^{1/2})^6 = x^{(\frac{1}{2})(6)} = x^3$$

Adding/Subtracting Exponential Terms: Add the scale factor. (*this can only be done if the terms are the same dimensions)

$$3x^3 + \underline{\underline{4x^2}} - \underline{\underline{\cancel{7y^2x}}} + \underline{\underline{3x^2}} - 8yx^2 \; \underline{\underline{+\cancel{7y^2x}}}$$

$$3x^3 + 7x^2 - 8yx^2$$

Remember Radicals are just exponents represented in a different way. Feel free to switch between these two forms.

Ex:

$\boxed{2l}$

$$x^7 = -128$$

$$x = (-128)^{1/7}$$

$$\underline{\underline{x = -2}}$$

$\boxed{3i}$

$$x^6 = 3$$

$$x = \pm\sqrt[6]{3} \quad \text{or} \quad \pm 3^{1/6}$$

$\boxed{4j}$

$$\sqrt{x^6y^4} = (x^6y^4)^{1/2}$$

$$= x^{6/2}y^{4/2}$$

$$= \underline{\underline{x^3y^2}}$$

$\boxed{6a}$

$$\sqrt{(x-2)^2}$$

$$= [(x-2)^2]^{1/2}$$

$$= (x-2)^{2/2}$$

$$= \underline{\underline{x-2}}$$

$\boxed{6j}$ $\sqrt{x^2-2x+1}$

$= \sqrt{(x-1)^2}$

$= \underline{\underline{x-1}}$

x^2-2x+1

$(x-1)(x-1)$

$(x-1)^2$

	+	×
	−2	+1
1+1 =	2	1 × 1
−1+−1 =	−2	−1 × −1

$\boxed{7f}$ $\sqrt[4]{\sqrt[3]{\sqrt{xy^2z^3}}}$ $= \left(\left(\left(xy^2z^3\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}} = \left(xy^2z^3\right)^{\frac{1}{24}} = \sqrt[24]{xy^2z^3}$

$= \left(\left(x^{\frac{1}{2}}\,y\,z^{3/2}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}$

$= \left(x^{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\,y^{\frac{1}{3}}\,z^{\left(\frac{3}{2}\right)\left(\frac{1}{3}\right)}\right)^{\frac{1}{4}}$

$= \left(x^{\frac{1}{6}}\,y^{\frac{1}{3}}\,z^{\frac{1}{2}}\right)^{1/4}$

$= x^{\left(\frac{1}{6}\right)\left(\frac{1}{4}\right)}\,y^{\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)}\,z^{\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)}$

$= x^{\frac{1}{24}}\,y^{\frac{1}{12}}\,z^{\frac{1}{8}}$ $\underline{\text{or}}$ $\sqrt[24]{x}\,\sqrt[12]{y}\,\sqrt[8]{z}$