Restrictions on Radical Equations

Question: What are we not allowed to do in Math?

There are two seemingly logical actions we take in math that are not allowed... (At least with real numbers). Thus we have created some rules around what we can and cannot do with real numbers.

Rules   *Even*

1. **Taking the Square Root of a Negative Number**
2. **Dividing by zero**

Some equations are thus, given restrictions on their variables due to these two rules.

Example: What values of **X** give you a square root of a negative number or a division by zero?

Restriction

① $\sqrt{4x-5}$ $\boxed{x \geqslant \frac{5}{4}}$

$4x - 5 \geqslant 0$
$\quad +5 \quad +5$
$\frac{4x}{4} \geqslant \frac{5}{4}$

② $\sqrt{x+6} - \sqrt{6-x}$

$x+6 \geqslant 0 \qquad 6-x \geqslant 0$
$-6 \quad -6 \qquad +x \quad +x$
$x \geqslant -6 \qquad 6 \geqslant x$
$\qquad\qquad\qquad x \leq 6$

★ Review

$7 - 3x \geqslant 1$
$-7 \qquad\qquad -7$

$\frac{-3x}{-3} \geqslant \frac{-6}{-3}$

$x \leqslant 2$

★ If you divide or multiply by a negative number you must flip your inequality

③ $\sqrt{\dfrac{6}{x-2}}$

rule #1   *rule #2*

$x - 2 \geqslant 0 \qquad x - 2 \neq 0$
$+2 \quad +2 \qquad\quad x \neq 2$
$x \geqslant 2$

$\boxed{x > 2}$

④ $\sqrt{\dfrac{(x-1)(x+3)}{(x-1)}}$ $x - 1 \neq 0$

$\sqrt{x+3} \quad ; \quad x \neq 1$

$x + 3 \geqslant 0$

$\boxed{x \geqslant -3 \quad ; \quad x \neq 1}$

Solve the Following Equations. (Remember to check the restrictions)

$\sqrt{x+1} = x - 1$

★ $x + 1 \geqslant 0$
$\boxed{x \geqslant -1}$

$(\sqrt{x+1})^2 = (x-1)^2$

$x + 1 = x^2 - 2x + 1$
$\quad -1 \qquad\qquad -1$
$-x \qquad\qquad -x$

$0 = x^2 - 3x$

$0 = x(x-3)$

$\underline{x = 0} \; ✓ \quad | \quad x - 3 = 0$
$\qquad\qquad\qquad \underline{x = 3} \; ✓$

$\sqrt{3-3x} = \sqrt{3x+2}$

$3 - 3x \geqslant 0 \qquad 3x + 2 \geqslant 0$
$-3x \geqslant -3 \qquad \boxed{x \geqslant -\frac{2}{3}}$
$\overline{-3} \quad \overline{-3}$
$\boxed{x \leq 1}$

(number line) $-\frac{2}{3} \quad 0 \qquad\quad 1$

$\sqrt{3-3x}^2 = \sqrt{3x+2}^2$

$3 - 3x = 3x + 2$
$-2 +3x \qquad +3x \quad -2$

$\frac{1}{6} = \frac{6x}{6}$

$\boxed{\frac{1}{6} = x}$ ✓

Exploring Products

What do you get when you multiply:
a negative by a negative? $+$
a positive by a positive? $+$
a negative by a positive? $-$
a positive by a negative? $-$

For what values of **X** is the following positive? Negative?

$$x^2 - 12x + 20$$

$$(x-10)(x-2) \quad \text{Product}$$

$$\begin{array}{c|c} -12 & 20 \\ \hline & -10x - 2 \end{array}$$

	$(x-10)$	$(x-2)$	Product
$x > 10$	$+$	$+$	$+$
$10 < x < 2$	$-$	$+$	$-$
$x < 2$	$-$	$-$	$+$

What are the restrictions on the following Expression

$$\sqrt{x^2 - 12x + 20}$$

$$\sqrt{(x-10)(x-2)}$$

$$x - 10 \neq 0 \quad | \quad x - 2 = 0$$
$$x = 10 \quad \quad x = 2$$

	$x-10$	$x-2$	$(x-10)(x-2)$	
$\boxed{x > 10}$	$+$	$+$	$+$	✓
$2 < x < 10$	$-$	$+$	$-$	✗
$\boxed{x < 2}$	$-$	$-$	$+$	✓

restrictions $x \geq 10$ and $x \leq 2$