Vectors

January 31, 2017 9:02 PM

In physics there are two different types of variables. Vectors and Scalars.

Vectors: a variable made up of magnitude and direction

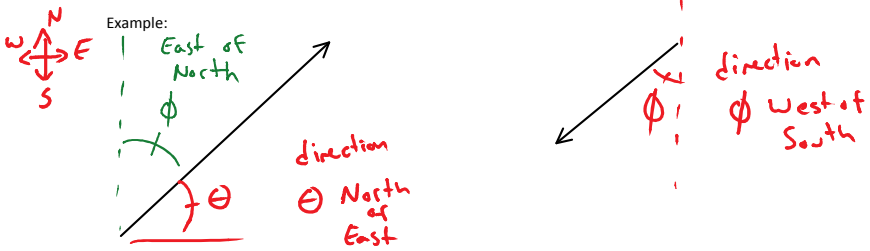
Example: Mr. Horncastle was going 100km/hr north this morning to work.

Scalar: a variable made up of only magnitude

Example: That space monkey has a max speed of 500,000m/s

Visually: We represent a vector as an arrow. The arrow points in the direction and has a length proportional to it's magnitude.

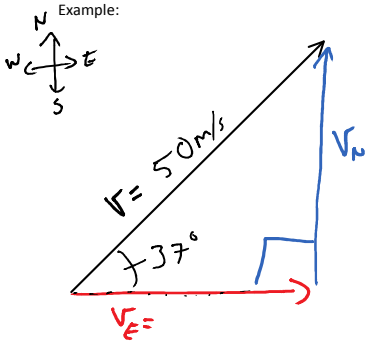
Example:



Cartesian Coordinate system

For the most part we work in Cartesian coordinate systems (X,Y,Z). Forces, accelerations, etc. We can break up vectors into these separate coordinates in order to simplify the math. In order to break up a vector into these coordinates we simply use our Sin, Cos, and Tan functions and the Pythagoras Theorem.

Example:



A pig travels at 50m/s at 37° North of east. Break this Vector up into it's North east components.

$\sin\theta = \frac{Opp}{Hyp}$ $\sin 37 = \frac{V_N}{V} = \frac{V_N}{50}$

$50\sin 37 = V_N$

$30.1 m/s = V_N$

$30 m/s = V_N$

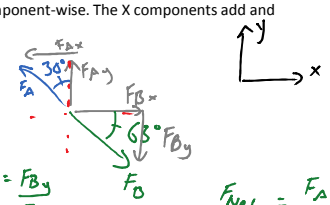
$\cos\theta = \frac{Adj}{Hyp}$ $V_E = 40 m/s$

$\cos 37 = \frac{V_E}{50}$

$50\cos 37 = V_E$

Adding and Subtracting vectors - Mathematically

In order to add and subtract vectors add together component-wise. The X components add and subtract together separately from the Y components.



Vector A : 10N 30° West of North

Vector B : 15N 63° South of East

$\sin 30 = \frac{F_{Ax}}{F_A}$

$10\sin 30 = F_{Ax}$

$-5N = F_{Ax}$

Negative because it goes left

$\cos 30 = \frac{F_{Ay}}{F_A}$

$10\cos 30 = F_{Ay}$

$8.66N = F_{Ay}$

$\sin 63 = \frac{F_{By}}{F_B}$

$15\sin 63 = F_{By}$

$-13.4N = F_{By}$

Negative because it goes down

$\cos 63 = \frac{F_{Bx}}{F_B}$

$15\cos 63 = F_{Bx}$

$6.81N = F_{Bx}$

$F_{Net_x} = F_{Ax} + F_{Bx}$

$= -5 + 6.81$

$F_{Net_x} = 1.81N$ right or East

$F_{Net_y} = F_{Ay} + F_{By}$

$= 8.66 - 13.4$

$F_{Net_y} = -4.74$

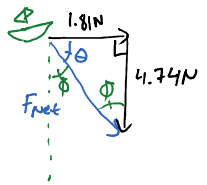
$F_{Net_y} = 4.74N$ down or South

Combine the Net Components back together

$8.66N = F_{Ay}$ $\quad 6.81N = F_{Bx}$ $\quad F_{Net_y} = 4.74N$ down or South

Combine the Net Components back together

Boat



$F_{Net}^2 = 1.81^2 + 4.74^2$

$F_{Net} = \sqrt{1.81^2 + 4.74^2}$

$F_{Net} = 5.07N$

$\tan\theta = \frac{4.74}{1.81}$

$\theta = \tan^{-1}\left[\frac{4.74}{1.81}\right]$

$\theta = 69°$

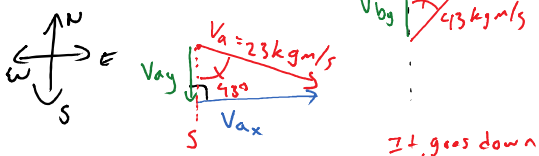
$F_{Net} = 5.07N$ @ $69°$ South of East

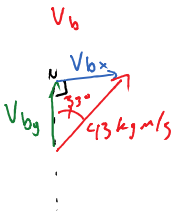
$F_{Net} = 5.07N$ @ $21°$ East of South

You try: Break the following vectors up into their components then add them together.
Find what Va + Vb equals. (Magnitude and Direction)

Vector A: 23kgm/s at 43° east of south
Vector B: 43kgm/s at 33° east of north

Step 1: Draw them





$V_{bx} + V_{ax} = 23.4 + 15.7$
$= 39.1 kgm/s$

$V_{by} + V_{ay} = 36.1 + (-16.8)$
$= 19.3 kgm/s$

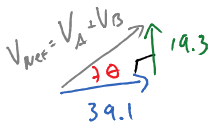
If it goes down

$V_{ay} = V_a \cos\theta = 23\cos 43 = -16.8 kgm/s$

$V_{ax} = V_a \sin\theta = 23\sin 43 = 15.7 kgm/s$

$V_{by} = V_b \cos\theta = 43\cos 33 = 36.1 kgm/s$

$V_{bx} = V_b \sin\theta = 43\sin 33 = 23.4 kgm/s$

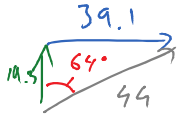


$\tan\theta = \frac{19.3}{39.1}$

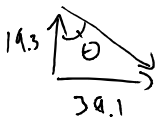
$\theta = 26°$

$V_{Net}^2 = 19.3^2 + 39.1^2$

$V_{Net} = 44 kgm/s$ @ $26°$ North of East ✓

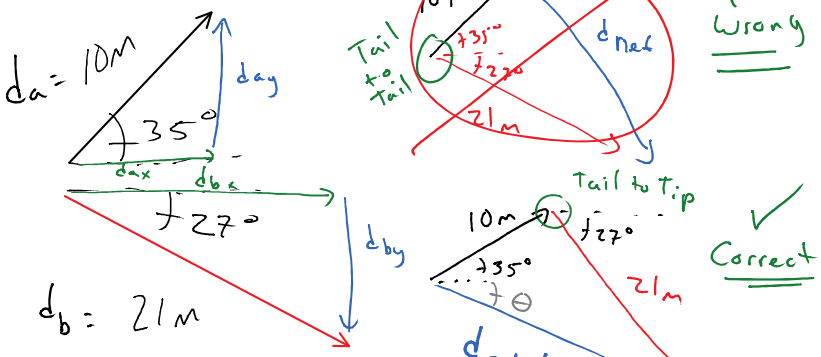


44 kgm/s @ 64° East of North ✓

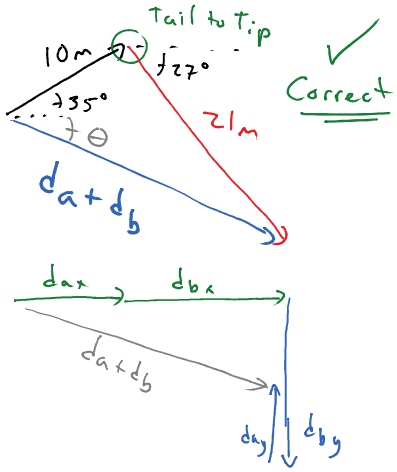


Adding and Subtracting vectors - Visually

Too add vectors visually, you simply connect the arrows tail to tip. Then the resultant vector can be drawn from the original tail to the final tip.



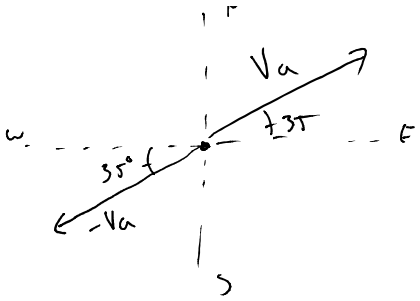
Wrong ✗

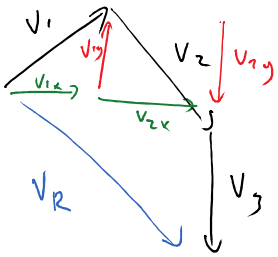


Correct ✓

Subtracting Vectors

When we subtract vectors we do the same thing as adding them, with the exception of reversing the direction. So a direction of 35° North of East will turn into 35° South of west.


N
V_a
± 35
W
E
35°
$-V_a$
S


V_1
V_{1y}
V_2
V_{2y}
V_{1x}
V_{2x}
V_R
V_3