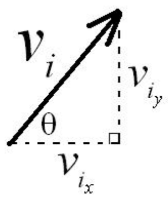
Kinematics 12 - Basic / Multi-angle / Complex

September 8, 2015 1:40 PM

Welcome to Kinematics in 2D! It's just like 1D kinematics only now you have twice as many variables, angles and you have to use Pythagoras and other weird math stuff.



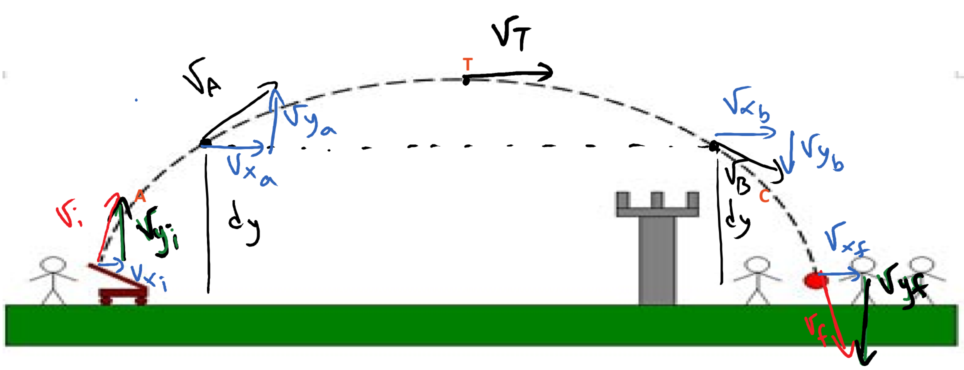
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} \sin 2\alpha &= 2\sin\alpha\cos\alpha \\ \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= 2\cos^2\alpha - 1 \\ &= 1 - 2\sin^2\alpha \\ \tan 2\alpha &= \frac{2\tan\alpha}{1 - \tan^2\alpha} \end{aligned}$$

Projectile Motion Problems

The development of Newtonian Mechanics (1600's) brought about the understanding of kinematics and a deeper understanding of projectile motion. This made scientists and mathematicians very popular as they could greatly improve the construction and function of various weapons.

Key Points in the Parabola



Due to Symmetry: $V_{y_i} = -V_{y_f}$ At the Top (T)

Due to Symmetry: $V_{y_i} = -V_{y_f}$

$V_{x_i} = V_{x_f}$

$V_{x_a} = V_{x_b}$

$V_{y_a} = -V_{y_b}$

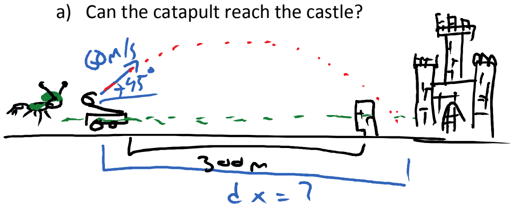
At the Top (T)

$V_{y_T} = 0\,m/s$

$V_T = V_{x_T}$

Basic Problem: An Army of Highly intelligent Carpenter Ants want to lay siege on a Castle. The Ants need to position their catapult 300m away from the walls in order to keep it out of range of the arrows. The catapults best shot is 60m/s at an angle of 45º to the horizontal.

a) Can the catapult reach the castle?



$V_i = 60\,m/s$

$V_{y_i} = 60\sin 45$

$V_{x_i} = 60\cos 45$

Vertical

$V_{y_i} = 60\sin 45$

$V_{y_f} = -60\sin 45$ – due to Symmetry

$d_y = 0\,m$ – it's Vertical Position is the Same initially as it is finally.

$a = -9.8\,m/s^2$

$t =$

$V_f = V_i + at$

$-60\sin 45 = 60\sin 45 - 9.8t$

$9.8t = 60\sin 45 + 60\sin 45$

$\frac{9.8t}{9.8} = \frac{120\sin 45}{9.8}$

$t = \frac{120\sin 45}{9.8}$

$t = 8.66\,sec$

Horizontal

$V_x = 60\cos 45$

$d_x =$

$t = 8.66\,sec$

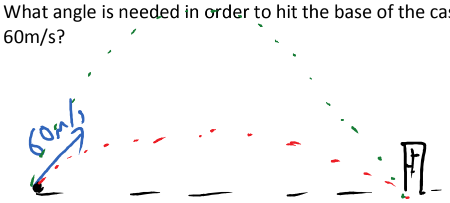
$d_x = V_x t$

$d_x = (60\cos 45)\left(\frac{120\sin 45}{9.8}\right)$

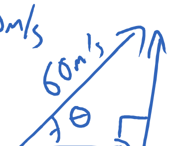
$d_x = 367\,m$

Yes the ants will be able to hit the wall.

b) What angle is needed in order to hit the base of the castle walls with a shot that has a speed of 60m/s?



$V_i = 60\,m/s$



$V_{y_i} = 60\sin\theta$

$V_x = 60\cos\theta$

Vertical

$V_{y_i} = 60\sin\theta$

$V_{y_f} = -60\sin\theta$

$a = -9.8\,m/s^2$

$d_y = 0\,m$

$t =$

$V_f^2 = V_i^2 + 2ad$

$(-60\sin\theta)^2 = (60\sin\theta)^2 + 2(9.8)(0)$

$\cancel{60^2}\cancel{\sin^2\theta} = \cancel{60^2}\cancel{\sin^2\theta}$

$1 = 1$

wrong...

Horizontal

$V_x = 60\cos\theta$

$d_x = 300\,m$

$t =$

$d_y = 0\,m$

$t =$

$1 = 1$

wrong info

Same times

$t =$

$t = \frac{d_x}{V_x} = \frac{300}{60\cos\theta}$

$$V_{y_f} = V_{y_i} + at$$

$$-60\sin\theta = 60\sin\theta - 9.8t$$

Sub in

$\boxed{t = \frac{300}{60\cos\theta}}$

$$-120\sin\theta = -9.8\left(\frac{300}{60\cos\theta}\right)$$

$$\sin\theta\cos\theta = \frac{(-9.8)(300)}{(60)(-120)}$$

Trig identity

$$2\sin\theta\cos\theta = \sin(2\theta)$$

apply identity

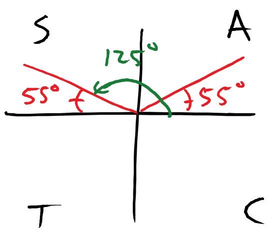
$$2\sin\theta\cos\theta = \frac{(2)(-9.8)(300)}{(60)(-120)}$$

$$\sin(2\theta) = 0.8166$$

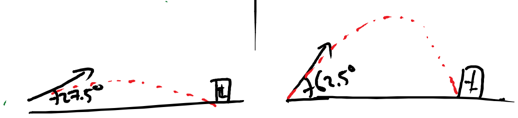
$$2\theta = \sin^{-1}[0.8166]$$

$$2\theta = 55°, 125°$$

$180 - 55° = 125°$



$2\theta = 55°$	$2\theta = 125°$
$\theta = 27.5°$	$\theta = 62.5°$



One of the most famous Basketball players of all time is Michael "Air" Jordan. He got this name due to his extraordinary hang time. Below is a famous picture of Michael dunking from the free throw line.

In this shot Michael rose to a maximum height of 3.5m. The Basketball hoop is 3m off the floor and the free throw line is 4.5m away. What was his initial velocity?

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