Momentum in 1D

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Momentum of a particle is the product of its mass and its velocity. As velocity is a vector quantity, so momentum itself is also a vector (i.e. it has both magnitude and direction) It has the same direction as the direction of the object in motion.

Momentum

Symbol: $\vec{p}$

SI Unit: kilogram meter per second (kg x m /s direction) (or Newton Seconds N·s)

$$\vec{p} = m\vec{v}$$

A heavy object moving slowly can have the same momentum as a light object moving quickly.

Ex 1: Compare the momentum of a 0.1 kg bullet moving at 1000 m/s and a 60 kg student moving at 1.7 m/s (6 km/h)

Bullet: $p = mv = (0.1)(1000)$, $p = 100$ kg m/s

Student: $p = (60)(1.7)$, $p = 102$ kg m/s

Ex 2: How fast does a 200g baseball need to be moving to have the equivalent momentum as a 5 kg bowling ball moving 8 m/s down a bowling lane? (200g = 0.2 kg)

Baseball: $p = mv$, $\frac{40}{0.2} = \frac{(0.2)(v)}{0.2}$, $200\text{ m/s} = v$

Bowling Ball: $p = mv = (5)(8)$, $p = 40$ kg m/s

To change the velocity of an object we need to apply a force to it for a period of time. This will give the object an acceleration, but that acceleration will be dependent on the force applied and the mass of the object (F=ma).

What we find is that heavy objects are harder to stop or move then lighter ones.

Impulse

Impulse is the force exerted on an object over a period of time.

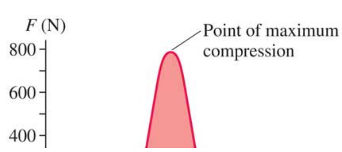
$$Imp = F \cdot t$$

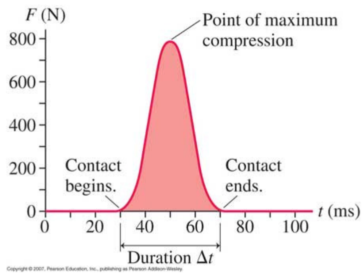
$Imp = P_f - P_i$

Units: N·s, kg m/s

Graphically the impulsive force may look like the graph below;

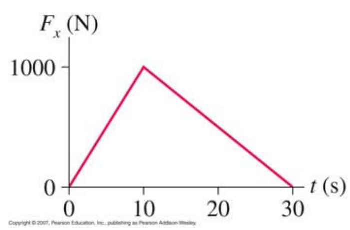
Impulse = Area under a Force vs. time Graph





Impulse = Area Graph

Far in space, where gravity is negligible, a 425 kg rocket traveling at 75 m/s fires its engine. The figure shows the thrust force as a function of time. The mass lost by the rocket during these 30.0 s is negligible.
a. What impulse does the engine impart to the rocket?
b. At what time does the rocket reach it maximum speed? What is the maximum speed?



a) Impulse = Area under Curve

$= \frac{1}{2} bh$

$= \frac{1}{2}(30)(1000)$

Impulse = 15 000 N.S

b) It is accelerating from t = 0s to t = 30s

Kinematics

$V_f = V_i + at$

$V_f - V_i = at$

Sub →

$Imp = F \cdot t$

$= mat$ $F = ma$

$= m(at)$

$= m(V_f - V_i)$

$Imp = mV_f - mV_i$

$15000 = 425 V_f - (425)(75)$

$\frac{15000 + (425)(75)}{425} = V_f$

$110 \text{ m/s} = V_f$

30 seconds later

From the problem above:

$$Imp = \Delta p$$
$$Imp = m(\vec{v_f} - \vec{v_i})$$

Combining the equation of impulse before with the one above we get:

$$\vec{F} \cdot t = m(\vec{v_f} - \vec{v_i})$$

Ex 1: a) What impulse is needed to make a 60 kg person moving at 5 m/s to end up moving at 2 m/s?

$Imp = F \cdot t$

$Imp = m(V_f - V_i)$

$Impulse = m(V_f - V_i)$

$= 60(2 - 5)$

$= 60(-3)$

$= -180 \text{ kg m/s}$

b) If the impulse used a force of -20N. How long would this force need to be incontact with the person?

Impulse = F·t
-180 = -20·t
-180/-20 = t
9 s = t

c) What force would need to be used to change this persons velocity in 0.5s?

Impulse = F·t
-180 = F (0.5)
-180/0.5 = F
-360 N = F

2. An impulse of -25 kgm/s is imparted upon a 2 kg object moving at 3m/s.
what is its new velocity?

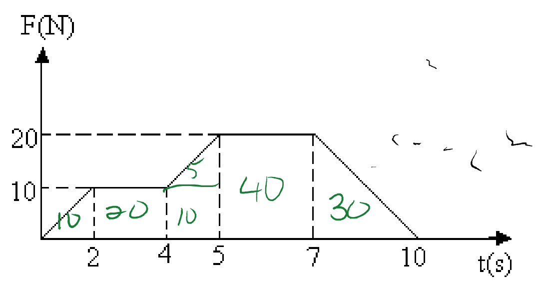
$Imp = m(V_f - V_i)$
$-25/2 = 2(V_f - 3)/2$
$-12.5 = V_f - 3$
+3 +3
$-9.5 \ m/s = V_f$

- Burning Bus (4)
- Sweater Fire (10)
- Bears (short) (1)

doors

Impulse Graphs - The area under a force vs. time graph give us the impulse.

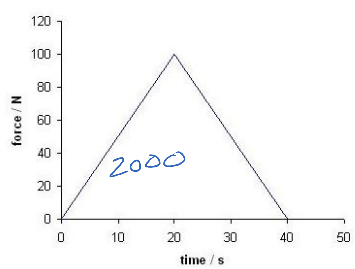
A) A 20 kg object experiences an impulse from the graph above, from 10 m/s
what is the new velocity?



Imp = F·t
= Area under the curve
Imp = 115 N·S

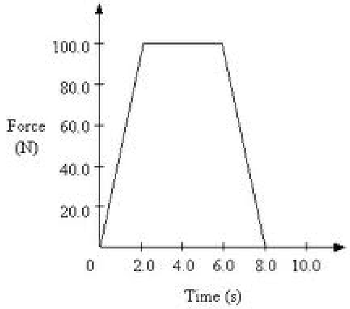
$115 = m(V_f - V_i)$
$115 = 20(V_f - 10)$
$115/20 + 10 = V_f$
$V_f = 15.75 \ m/s$

What is the final velocity of the 5 kg block, initially at rest, after it has had an impulse imparted(applied) on it. Use the graph to help you.



Imp = Area under Curve
= 2000 N·S
$Imp = m(V_f - V_i)$
$2000/5 = 5(V_f)/5$
$400 \ m/s = V_f$

If the mass of the object is 3.0 kg, what is its final velocity over the 8.0 s time period if it starts from rest?



Conservation of Momentum

Closed System: Does not exchange any matter with its surroundings and no external forces act on the system.

Conservation of Momentum tells us that in a closed system the total initial momentum equals the total final momentum. In other words no momentum is lost.

$$\vec{p_i} = \vec{p_f}$$

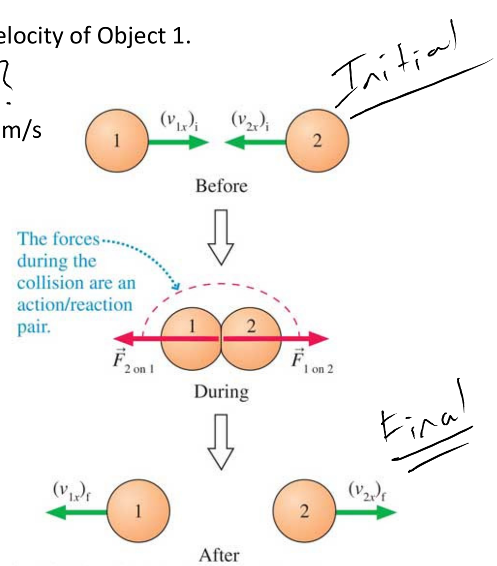
In this section we will explore what happens to the momentum of a system in three types of collisions.

Collisions (Non-stick)	totally Elastic
Collisions (Stick)	inelastic
Explosions (break apart)	inelastic

Below is a totally elastic collisions. Find the Velocity of Object 1.

Given:

M_1 = 4 kg	V_{1i} = 8 m/s	V_{1f} = ?
M_2 = 3 kg	V_{2i} = -8 m/s	V_{2f} = 14 m/s



$P_i = m_1V_{1i} + m_2V_{2i}$
$= 4(8) + 3(-8)$
$P_i = 8$ kg m/s

$P_f = m_1V_{1f} + m_2V_{2f}$
$= 4(V_{1f}) + 3(14)$
$P_f = 4V_{1f} + 42$

$P_i = P_f$
$8 = 4V_{1f} + 42$
-42

$V_{1f} = \frac{-34}{4}$
$V_{1f} = -8.5$ m/s

$P_i = P_f$

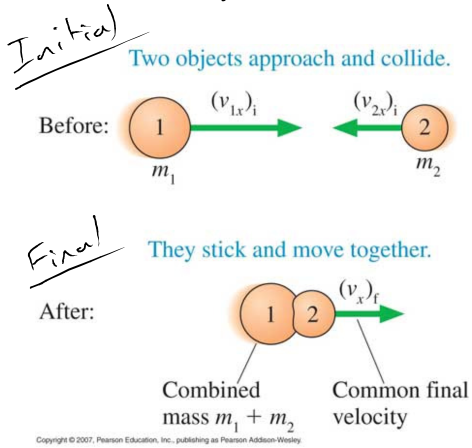
$8 = 4V_{1f} + 42$

$-42 \quad -42$

$-34 = 4V_{1f} \rightarrow V_{1f} = \frac{-34}{4}$

$V_{1f} = -8.5\,m/s$

Below is a inelastic collision where the two objects collide and stick together. Find the mass of Object 2.



$M_1 = 5.0kg$
$V_{1i} = 6.0m/s$
$V_{2i} = -4.0m/s$
$V_f = 1.5m/s$

$P_i = m_1V_{1i} + m_2V_{2i}$

$P_i = 5(6) + m_2(-4)$

$P_i = 30 - 4m_2$

$P_f = (m_1 + m_2)V_f$

$= (5 + m_2)(1.5)$

$P_f = 7.5 + 1.5m_2$

$P_i = P_f$

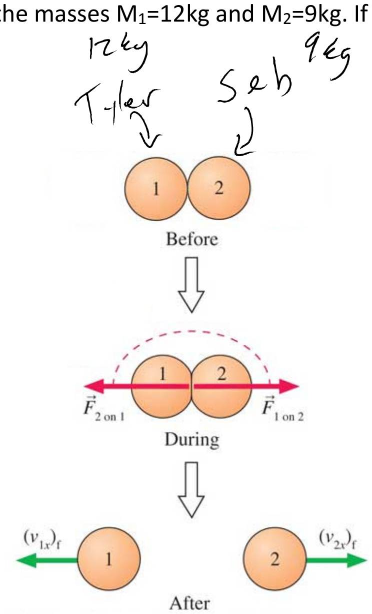
$30 - 4m_2 = 7.5 + 1.5m_2$

$30 - 7.5 = 1.5m_2 + 4m_2$

$\frac{22.5}{5.5} = \frac{5.5m_2}{5.5}$

$m_2 = 4.1kg$

Below is an explosion. Find the relative velocities of the masses M_1=12kg and M_2=9kg. If the initial momentum of the system is 0.



$P_i = 0$

$P_f = m_1V_{1f} + m_2V_{2f}$

$P_f = 12V_{1f} + 9V_{2f}$

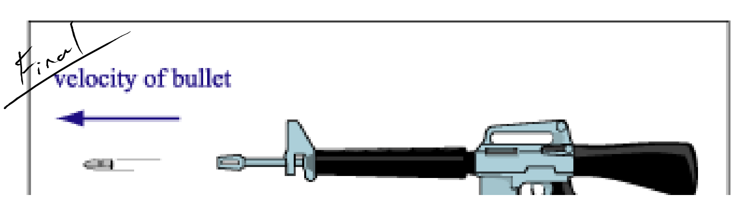
$P_i = P_f$

$0 = 12V_{1f} + 9V_{2f}$

$\frac{-9V_{2f}}{-9} = \frac{12V_{1f}}{-9}$

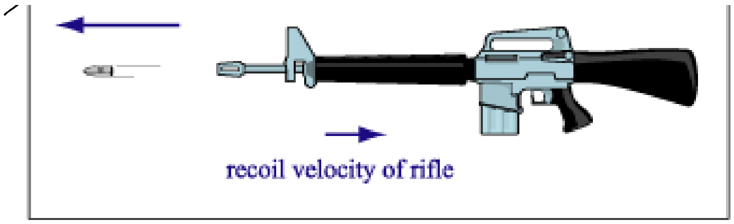
$V_{2f} = -\frac{4}{3}V_{1f}$

Find the recoil velocity of the 10kg gun if it fires a 9.5g bullet out at a velocity of 900m/s. (Assume that before the gun is fired it is not moving)



$P_i = 0\,kg\,m/s$

$P_f = (0.0095)(900) + (10)V_f$



$P_f = (0.0095)(900) + (10)V_f$

$P_i = P_f$

$0 = (0.0095)(900) + 10V_f$

$V_f = \frac{(0.0095)(900)}{-10}$

$V_f = -0.855\ m/s$

The gun moves with a velocity of 0.855 m/s to the Right.

What if the 67kg person holding the gun puts their full weight behind it?

Same as before but add more mass to the gun i.e. $10 \to 77$

$0 = (0.0095)(900) + 77V_f$

$V_f = -\frac{(0.0095)(900)}{77}$

$V_f = -0.11\ m/s$

The ballistic pendulum was used to measure the speeds of bullets before electronic timing devises were developed. The version shown in the figure consists of a large block of wood of mass M = 5.4 kg, hanging from two long cords. A bullet of mass m = 9.5 g is fired into the block, coming quickly to rest. The block+bullet then swing upward, their center of mass rising a vertical distance h = 6.3 cm, with this information they were able to find the final velocity of the bullet pendulum system to be 1.11m/s. What was the initial velocity of the bullet.