Equilibrium

September 8, 2015 1:35 PM

Equilibrium occurs when an object is not accelerated and is not rotating.

In order for an object to be in equilibrium the following two conditions for equilibrium must be satisfied:

First Condition of Equilibrium is satisfied when the sum of the forces acting on an object equals zero in all directions, **ΣF = 0**.

$F_{Net_x} = 0$
$F_{Net_y} = 0$

Second Condition of Equilibrium is satisfied when the sum of all torques acting on an object about any axis perpendicular to the plane of the forces equals zero, **Στ = 0**

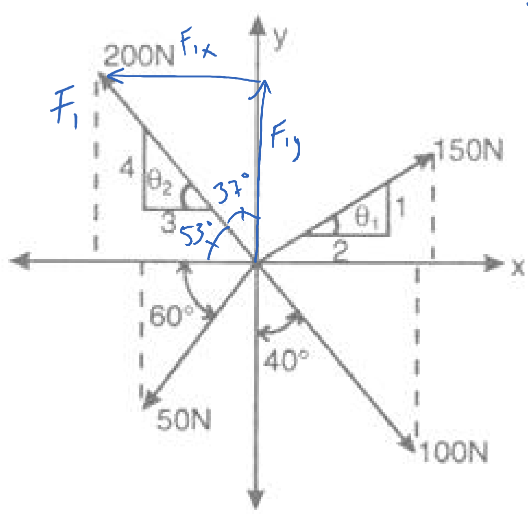
(Rotational Forces)

Static Equilibrium occurs when an object at equilibrium is at rest.

Dynamic Equilibrium occurs when an object at equilibrium is moving in a straight line at a constant speed.

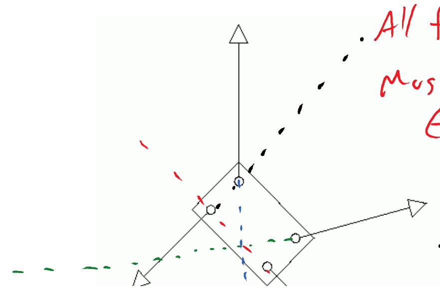
Concurrent Forces are aligned such that the line of action of each force acting on the object passes through a common point. If the forces are concurrent, only the first condition of equilibrium (**ΣF = 0**) is needed to solve the problem.

$F_{1x} = -200 \sin 37$
$F_{1y} = 200 \cos 37$

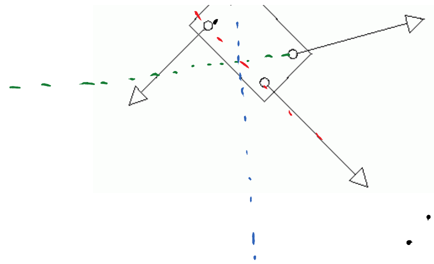


Non-concurrent Forces are aligned such that the line of action of each force does not pass through a common point. The result is a net torque which tends to cause the object to rotate. If the forces are non-concurrent, and yet the object is in equilibrium, then both conditions of equilibrium are required to solve the problem (**ΣF = 0** and **Στ = 0**)

All forces and Torques must balance for Equilibrium.



Forces do not



Forces do not originate from a common point

∴ Non-Concurrent Forces

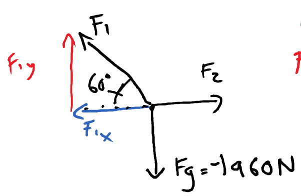
∴ Both laws must be satisfied

Problem Solving Skills

1. Draw an accurate diagram locating the forces acting on the object or systems of objects.
2. Draw a free body diagram locating the forces acting on the object(s) in question.
3. **Resolve each force vector into x and y components.**
4. Apply the first condition of equilibrium ($\Sigma F_x=0$, $\Sigma F_y=0$) and solve the problem.

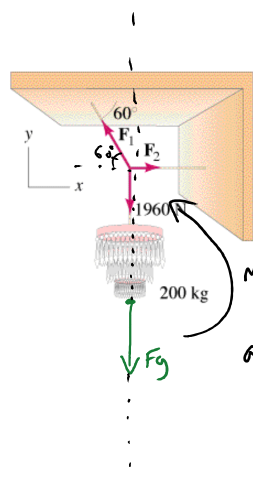
$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

1) Calculate the tension in the two cords used to support the 200 kg chandelier.



$F_{1x} = F_1 \cos 60$

$F_{1y} = F_1 \sin 60$



move Fg along the action line

$\Sigma F_x = 0$

$-F_{1x} + F_2 = 0$

$-F_1 \cos 60 + F_2 = 0$

$F_2 = +F_1 \cos 60$

$= +2263 \cos 60$

$\underline{\underline{F_2 = +1132\ N}}$

$\Sigma F_y = 0$

$F_{1y} + F_g = 0$

$F_1 \sin 60 - 1960 = 0$

$F_1 = \dfrac{1960}{\sin 60}$

$\underline{\underline{F_1 = 2263\ N}}$

2) Three forces are applied to a tree sapling, as shown below, to stabilize it. If F_1=282N and F_2=355N, find F_3 in magnitude and direction.

$\Sigma F_y = 0$

$F_{2x} = F_2 \cos 70 = -121\ N$

$F_{2y} = F_2 \sin 70 = 334\ N$

$\Sigma F_y = 0$

$F_{2y} + F_{3y} = 0$

$334 - F_3 \sin\theta = 0$

Eq1 $\quad 334 = F_3 \sin\theta$

$\Sigma F_x = 0$

$F_1 + F_{2x} + F_{3x} = 0$

$282 - 121 - F_3 \cos\theta = 0$

Eq2 $\quad 161 = F_3 \cos\theta$

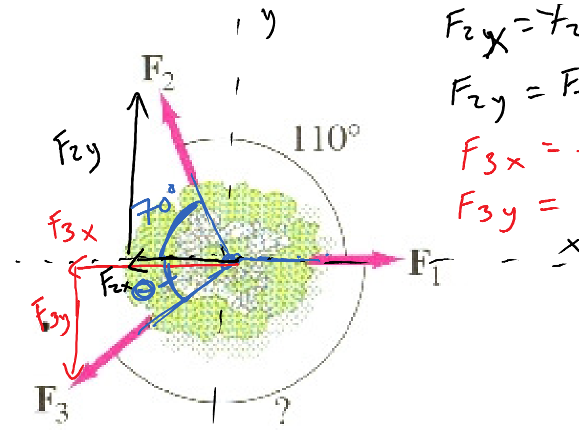


FIGURE 9-47 Problem 1.

$F_{2x} = -F_2 \cos 70 = -121\,N$

$F_{2y} = F_2 \sin 70 = 334\,N$

$F_{3x} = -F_3 \cos\theta$

$F_{3y} = -F_3 \sin\theta$

$\boxed{\dfrac{Eq1}{Eq2} \quad \text{divide equations}}$ or Substitute

$$\frac{334}{161} = \frac{\cancel{F_3}\sin\theta}{\cancel{F_3}\cos\theta}$$

$$\frac{334}{161} = \tan\theta$$

$$\tan^{-1}\left[\frac{334}{161}\right] = \theta$$

$\theta = 64°$

$334 = F_3 \sin(64)$

$F_3 = \dfrac{334}{\sin 64}$

$F_3 = 372N$

$\boxed{F_3 = 372N \text{ @ } 64° \text{ South of west}}$

Find the tension in the two wires supporting the traffic light shown below.

$\Sigma F_y = 0$

$T_{1y} + T_{2y} - F_g = 0$

$T_1 \sin 53 + T_2 \sin 37 - 294 = 0$

$\Sigma F_x = 0$

$T_{2x} - T_{1x} = 0$

$T_2 \cos 37 - T_1 \cos 53 = 0$

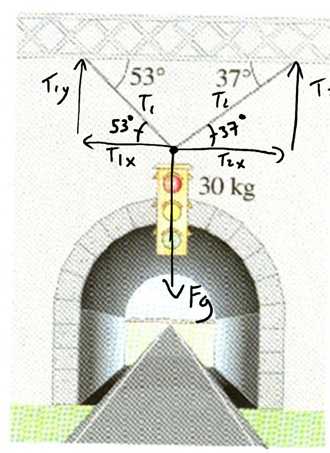
Substitute $\quad T_2 = \dfrac{T_1 \cos 53}{\cos 37}$

$T_1 \sin 53 + T_1 \dfrac{\cos 53 \sin 37}{\cos 37} - 294 = 0$

$T_1[\sin 53 + \cos 53 \tan 37] = 294$

$T_1 = \dfrac{294}{\sin 53 + \cos 53 \tan 37}$

$T_1 = 235N$

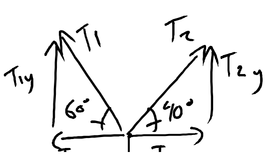


$T_2 = \dfrac{T_1 \cos 53}{\cos 37}$

$= \dfrac{235 \cos 53}{\cos 37}$

$T_2 = 177N$

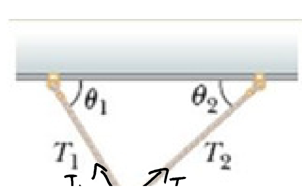
A bag of cement weighing 325 N hangs in equilibrium from three wires. Two of the wires make angles θ_1 =60.0° and θ_2 =40.0° with the horizontal. Assuming the system is in equilibrium, find the tensions T_1, T_2, and T_3 in the wires

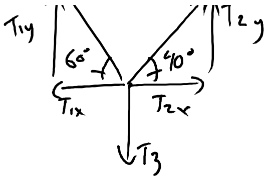


$\Sigma F_x = 0$

$T_{2x} - T_{1x} = 0$

$T_2 \cos 40 - T_1 \cos 60 = 0$





$T_{2x} - T_{1x} = 0$

$T_2 \cos 40 - T_1 \cos 60 = 0$

$T_2 = \frac{T_1 \cos 60}{\cos 40}$ Eq 1

Sub

$\Sigma F_y = 0$

$T_{1y} + T_{2y} - T_3 = 0$

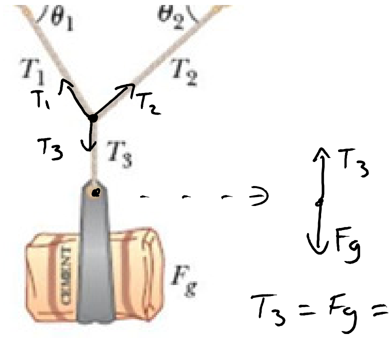
$T_1 \sin 60 + T_2 \sin 40 = 325$

$T_1 \sin 60 + T_1 \frac{\cos 60 \sin 40}{\cos 40} = 325$

$T_1 \left[\sin 60 + \cos 60 \tan 40 \right] = 325$

$T_1 = \frac{325}{\sin 60 + \cos 60 \tan 40}$

$T_1 = 253N$

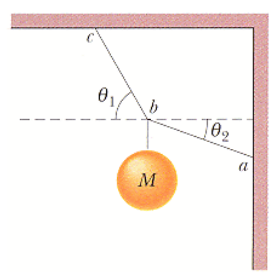


$T_3 = F_g = 325N$

$T_2 = \frac{253 \cos 60}{\cos 40}$

$T_2 = 165N$

The system in the figure below is in equilibrium. The angles are $\theta_1 = 58°$ and $\theta_2 = 15°$, and the ball has mass M = 2.3 kg.
(a) What is the tension in string ab?
(b) What is the tension in string bc?



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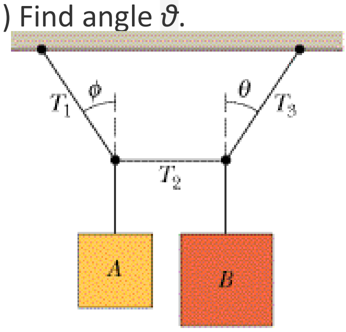
The system below is in equilibrium, with the string in the center exactly horizontal. Block *A* weighs 25 N, block *B* weighs 55 N, and angle is 36°.

(a) Find tension T_1.
(b) Find tension T_2.
(c) Find tension T_3.
(d) Find angle ϑ.



$T_1 \cos\phi = 25N$

$\Sigma F_y = 0$ $\quad T_1 \cos 36° = 25N$

$T_1 = \frac{25}{\cos 36°} = 30.9N$

$\Sigma F_x = 0$ $\quad T_1 \sin\phi = T_2$

$T_2 = 30.9(\sin 36°) = 18.16N$

$\Sigma F_x = 0$ $\quad T_3 \sin\theta = 18.16N$

$\Sigma F_y = 0$ $\quad T_3 \cos\theta = 55$ $\quad T_3 = \frac{55}{\cos\theta}$

$\frac{55}{\cos\theta} \cdot \sin\theta = 18.16$

$\tan\theta = \frac{18.16}{55} = 18.27°$

$T_3 = \frac{55}{\cos 18.27} = 57.9N$

Formulas and Identities

Tangent and Cotangent Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\qquad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\qquad$ $\sin\theta = \frac{1}{\csc\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\qquad$ $\cos\theta = \frac{1}{\sec\theta}$

$\cot\theta = \frac{1}{\tan\theta}$ $\qquad$ $\tan\theta = \frac{1}{\cot\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Even/Odd Formulas

$\sin(-\theta) = -\sin\theta$ $\qquad$ $\csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$ $\qquad$ $\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\qquad$ $\cot(-\theta) = -\cot\theta$

Periodic Formulas

If n is an integer.

$\sin(\theta + 2\pi n) = \sin\theta$ $\qquad$ $\csc(\theta + 2\pi n) = \csc\theta$

$\cos(\theta + 2\pi n) = \cos\theta$ $\qquad$ $\sec(\theta + 2\pi n) = \sec\theta$

$\tan(\theta + \pi n) = \tan\theta$ $\qquad$ $\cot(\theta + \pi n) = \cot\theta$

Double Angle Formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas

$\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$

$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$

$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$

Sum and Difference Formulas

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Product to Sum Formulas

$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$

$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$

$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$

$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]$

Sum to Product Formulas

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$

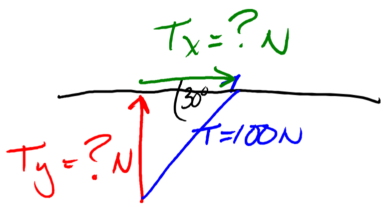
Cofunction Formulas

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\qquad$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\qquad$ $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$ $\qquad$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

A reminder: To find a component of tension:



$T_x = 100 \cos 30° = 86.6 N$

$T_y = 100 \sin 30° = 50 N$

But sometimes you have the component and you want to find the tension of the wire.