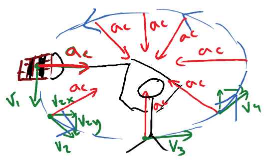
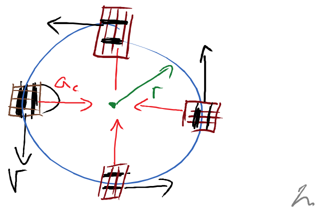
Circular Motion

September 8, 2015 1:36 PM

We have just finished learning about forces and torques. Remember torques are forces that want to cause a rotational movement. Now we will learn about this Circular Motion.

Imagine Spinning a bucket around your head.

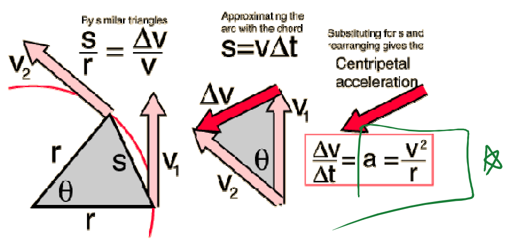




Circular Uniform Motion is caused by a force directed to the center of a circular path on an object. This force creates an acceleration towards the center of rotation. This acceleration is called centripetal acceleration which changes the direction of the velocity of vector.

The magnitude of the Velocity and Acceleration Vectors do not change, only the direction. (Constant Speed)

Deriving the Formula



Formula

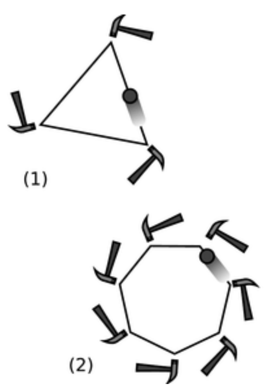
$$a_c = \frac{v^2}{r} \qquad v = \frac{2\pi r}{T} \qquad a_c = \frac{4\pi^2 r}{T^2}$$

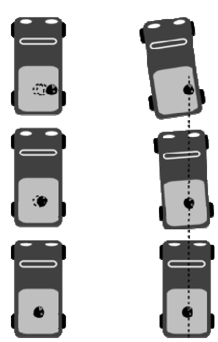
$2\pi r$ = Circumference

T = time for one complete Revolution

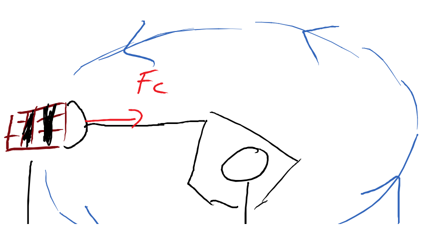
Period

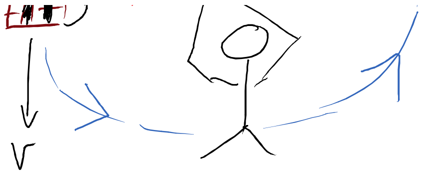
Newton's first law states that object in motion stay in motion (ie. If the object is moving in a straight line, it will continue to move in a straight line if no net Force is acting on it). What makes objects move in a curved path? A string pulls on an object swinging above your head or in front of you. Tires on a car keep the car on the road when cornering. Gravity keeps you in your seat as your go over a bump.





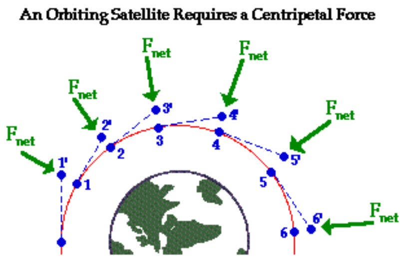
Consider that bucket of mass ***m*** tied to a string of length ***r*** and being whirled in a horizontal circular path above my head. The inertia of the bucket tends to maintain the straight line and the tension in the string exerts a force on the bucket to make it follow a circular path. This force is directed along the length of the string toward the center of the circle. This force is called **centripetal force.**





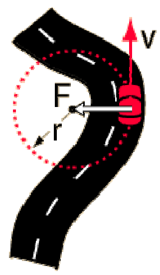
$$F_c = ma_c = \frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2}$$

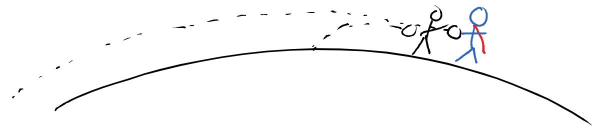
** The Term Centripetal indicates that the force is always directed towards the center of the circle.

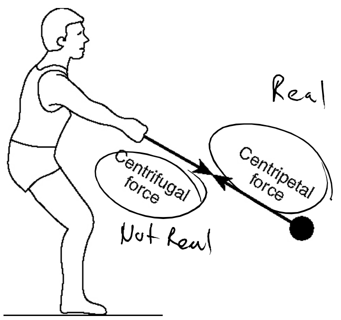


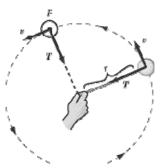
$$F_{centripetal} = m\frac{v^2}{r}$$

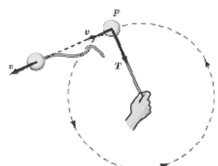
$\frac{v^2}{r}$ is the centripetal acceleration

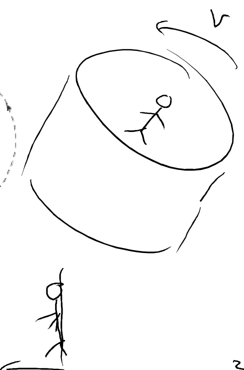








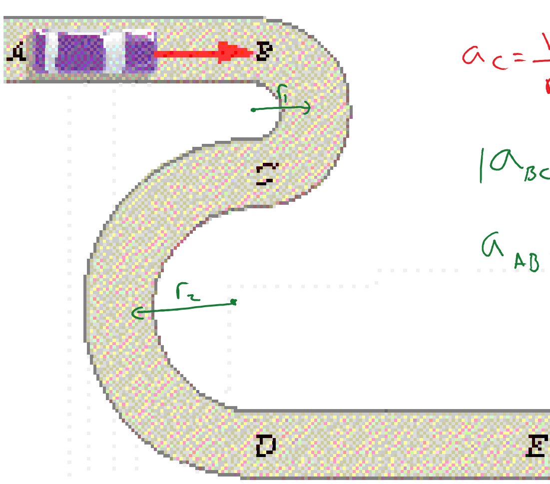




$$F_{Net} = F_N = m a_c = \frac{mv^2}{r}$$

** Centrifugal force is a force that is caused by Newtons 3rd law. It is NOT a real force but rather is the force we feel when we are in a circular motion frame of reference.

A car is traveling at a constant speed along the road ABCDE shown in the drawing. Sections AB and DE are straight. Rank the magnitude of the accelerations for each section from highest to lowest



$a_c = \frac{v^2}{r}$ ← To maximize a_c we need to minimize r

$|a_{BC}| > |a_{CD}|$ b/c $r_1 < r_2$

$a_{AB} = a_{DE} = 0$

Example: A toy car completes one lap around a circular track (distance of 200 m) in 25 s.
(a) What is the average speed?
(b) If the mass of the car is 1.5 kg, what is the magnitude of the centripetal force that keeps it in a circle?

$C = 200\,m \quad v = \frac{d_{Total}}{t_{Total}} = \frac{200}{25} \quad C = 2\pi r$

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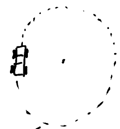


$C = 200\,m$

$T = 25\,sec$

$v = \frac{d_{Total}}{t_{Total}} = \frac{200}{25}$

$v_{average} = 8\,m/s$

$C = 2\pi r$

$\frac{200}{2\pi} = r$

$F_{net} = ma$

T in this problem stands for period

Fnet equation for Circular motion

$F_c = ma_c = \frac{m4\pi^2 r}{T^2} = \frac{(1.5)(4)(\pi^2)\left(\frac{200}{2\pi}\right)}{25^2}$

$\underline{F_c = 3.02\,N}$

Example: A 3 kg mass attached to a light string rotates in circular motion on a horizontal, frictionless table. The radius of the circle is 0.8 m, and the string can support a mass of 25 kg before breaking.
(a) What is the maximum speed the mass can have before the string breaks?

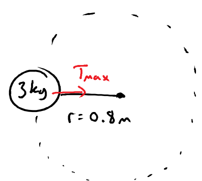
$F_{net} = \cancel{ma}$

$T - F_g = 0$

$T = F_g$

$T_{max} = (25)(9.8)$

$\underline{T_{max} = 245\,N}$



$F_{net_c} = ma_c$

$T_{max} = \frac{mv^2}{r}$

$245 = \frac{3v^2}{0.8}$

$\sqrt{\frac{245(0.8)}{3}} = v$

$\boxed{8.08\,m/s = v}$

T in this problem is for Tension

(b) What is the shortest period of rotation the object can have?

Now T is Period

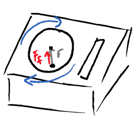
$T = \frac{2\pi r}{v} = \frac{2\pi(0.8)}{8.08}$

$\underline{T = 0.62\,sec}$

T : Period

F_T + T : Tension

Example: A coin is placed 30 cm from the center of a rotating, horizontal turntable. The coin is observed to slip when its speed is 50 cm/s.
(a) What provides the centripetal force when the coin is stationary relative to the turntable? Friction (Towards the center)
(b) What is the coefficient of static friction between the coin and the turntable?



$r = 0.3\,m$

$v_{max} = 0.5\,m/s$

$F_f = ma_c$

$\mu F_g = ma_c$

$\mu \cancel{m} g = \cancel{m} a_c$

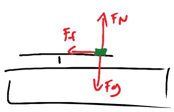
$\mu = \frac{a_c}{g}$

$a_c = \frac{v^2}{r}$

$\mu = \frac{v^2}{rg}$

$= \frac{(0.5)^2}{(0.3)(9.8)}$

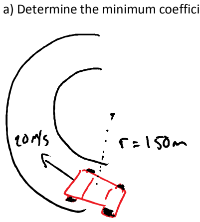
$\boxed{\mu = 0.085}$

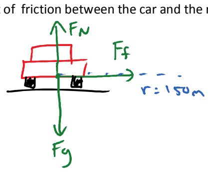


$F_N - F_g = m\cancel{a}$

$F_N = F_g$

Example: A highway curve has a radius of 150 m and is designed for a traffic speed of 20 m/s.
a) Determine the minimum coefficient of friction between the car and the road.





Vertical a=0

$F_N = F_g$

Circular

$F_c = ma_c$

$F_f = ma_c$

$\mu F_N = ma_c$

$\mu \cancel{m} g = \cancel{m} a_c$

$a_c = \frac{v^2}{r}$

$\boxed{\mu = \frac{v^2}{rg}} = \frac{20^2}{(150)(9.8)}$

$\boxed{\mu = 0.272}$

b) If the road's bend shortens down to 40m, what is the maximum speed you can take the curve?

$\mu = \frac{v^2}{rg}$

$0.272 = \frac{v^2}{(40)(9.8)}$

$\sqrt{(0.272)(40)(9.8)} = v$

$10\,m/s = v$

$37\,km/h = v$