Circular Motion - Non-uniform Motion

Remember from Last Class

$$F_c = ma_c = \frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2}$$

$$a_c = \frac{v^2}{r} \qquad v = \frac{2\pi r}{T} \qquad a_c = \frac{4\pi^2 r}{T^2}$$

Also remember from dynamics that:

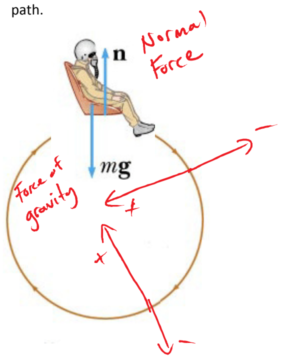
$$F_{net} = ma$$

Circular motion uses

$$F_{net} = ma_c$$

We define the + direction to be towards the center of the circle (ie. a_c is always positive)

Consider a jet doing a vertical arc in a circular path. Look at the force's on the pilot at the top of the path.



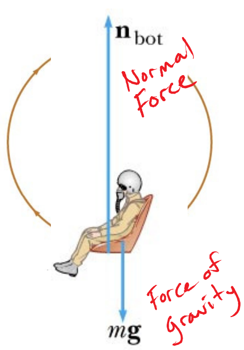
$$F_{Net} = ma$$

$$F_{Net} = \frac{mv^2}{r}$$

$$-F_N + F_g = \frac{mv^2}{r}$$

$$F_N = F_g - \frac{mv^2}{r}$$

Now consider the same jet doing the same vertical arc but this time look at the force's at the bottom of the path.

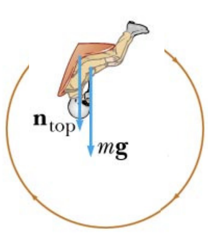


$$F_{Net} = \frac{mv^2}{r}$$

$$F_N - F_g = \frac{mv^2}{r}$$

$$F_N = F_g + \frac{mv^2}{r}$$

What if the jet was doing the arc inverted?



$$F_{Net} = \frac{mv^2}{r}$$

$$F_N + F_g = \frac{mv^2}{r}$$

$$F_N = \frac{mv^2}{r} - F_g$$

Top of the Arc

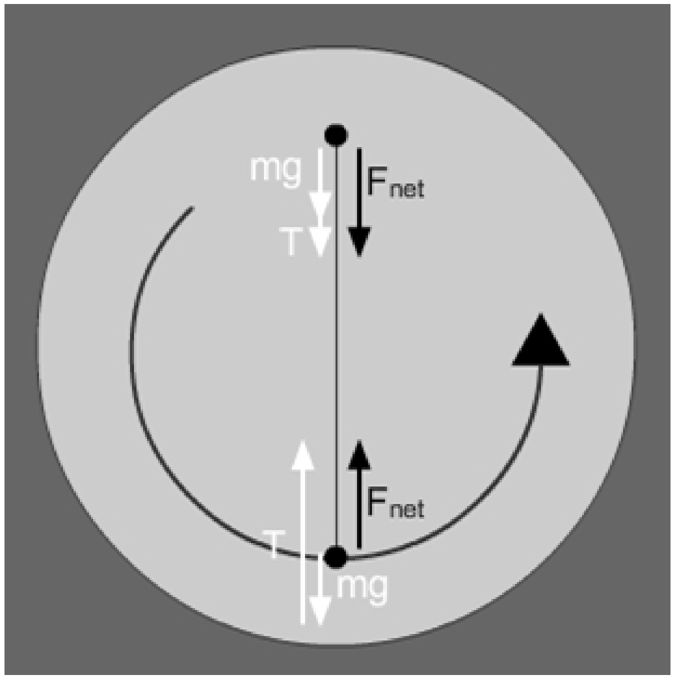
$$T + F_g = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r} - F_g$$

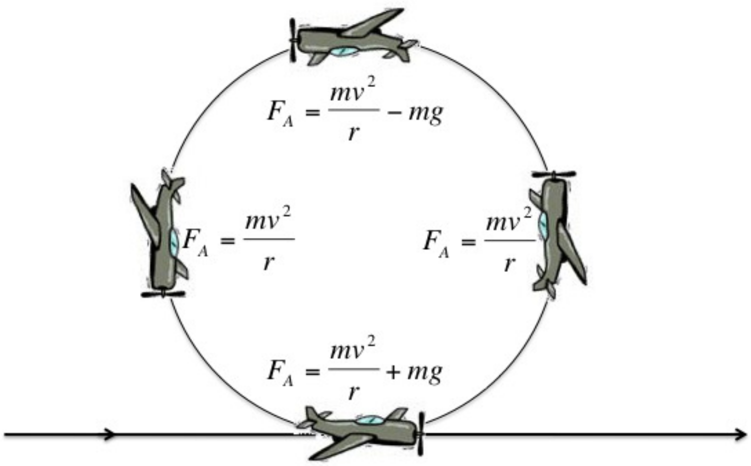
Bottom of Arc

$$T - F_g = \frac{mv^2}{r}$$

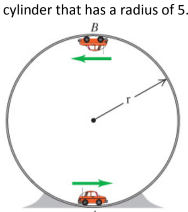
$$T = \frac{mv^2}{r} + F_g$$



☆ T is largest at the bottom of a circular path



A small remote-control car with a mass of 1.51 kg moves at a constant speed of v = 12.0 m/s in a vertical circle inside a hollow metal cylinder that has a radius of 5.00 m.



at A

F_N +

F_g -

Notice ☆ F_N is larger than F_g

$F_{Net} = ma_c$

$F_N - F_g = ma_c$

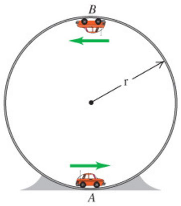
$F_N = \frac{mv^2}{r} + mg$

a) What is the magnitude of the normal force exerted on the car by the walls of the cylinder at point A (at the bottom of the vertical circle)?

$F_N = 1.51\left[\frac{12^2}{5} + 9.8\right]$

$\boxed{F_N = 58.3\ \text{N} \text{ into the Circle}}$

b. What is the magnitude of the normal force exerted on the car by the walls of the cylinder at point B (at the top of the vertical circle)?



at B

$F_N \downarrow \quad \downarrow F_g$

$+$

$F_{net} = ma$

$F_N + F_g = ma$

$F_N = \frac{mv^2}{r} - mg$

$= 1.51\left(\frac{12^2}{5} - 9.8\right)$

$F_N = 28.7\,N$

c. What is the minimum velocity needed to complete the loop?

This will happen when $F_N = 0$

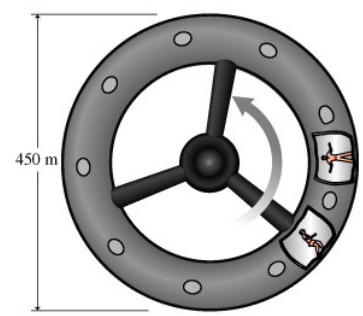
$F_{net} = ma_c$

$F_g = \frac{mv^2}{r}$

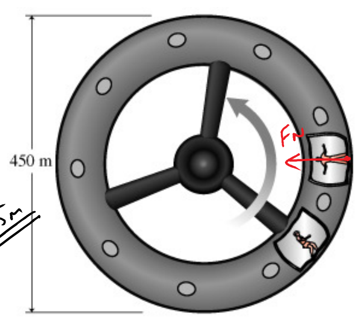
$\cancel{m}g = \frac{\cancel{m}v^2}{r}$

$\sqrt{gr} = v$

$\boxed{7\,m/s = v}$



A space station is in the shape of a hollow ring, 450 m in diameter.



$r = 225\,m$

At how many revolutions per minute should it rotate in order to simulate Earth's gravity, that is, so that the normal force on an astronaut at the outer edge would be the astronaut's weight on Earth?

$F_N = F_g$

$F_{net} = ma_c$

$F_N = ma_c$

$F_g = ma_c$

$\cancel{m}g = \cancel{m}a_c$

$g = a_c$

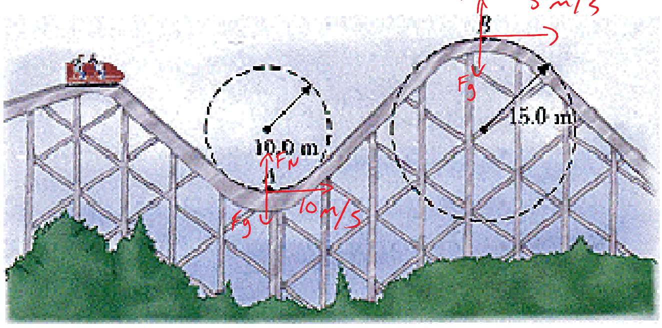
$g = \frac{4\pi^2 r}{T^2}$

$T^2 = \frac{4\pi^2 r}{g}$

$T = \sqrt{\frac{4\pi^2 r}{g}}$

$T = 30\ \text{sec/rev}$

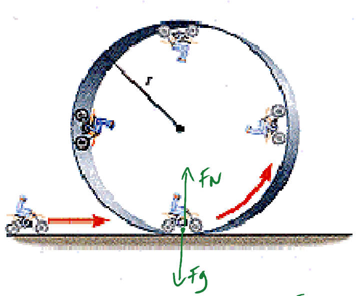
a) If the roller coaster is moving at 10 m/s at point A, what force does the 60 kg person feel from his/her seat?
b) If the roller coaster is moving at 5 m/s at point B, what force does the 60 kg person feel from his/her seat?

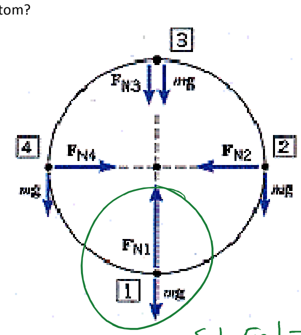


a)
$$F_{Net} = \frac{mv^2}{r}$$
$$F_N - F_g = \frac{mv^2}{r}$$
$$F_N = \frac{mv^2}{r} + F_g$$
$$= \frac{(60)(10)^2}{10} + (60)(9.8)$$
$$F_N = \underline{\underline{1188\,N}}$$

b)
$$F_{Net} = \frac{mv^2}{r}$$
$$F_g - F_N = \frac{mv^2}{r}$$
$$F_N = F_g - \frac{mv^2}{r}$$
$$= (60)(9.8) - \frac{(60)(5)^2}{15}$$
$$= \underline{\underline{488\,N}}$$

For the normal force below to have the same magnitude at all points on the vertical track, the stunt driver must adjust the speed to be different at different points. Suppose, for example, that the track has a radius of 2.8 m and that the driver goes past point 1 at the bottom with a speed of 14 m/s. What speed must she have at point 3, so that the normal force at the top has the same magnitude as it did at the bottom?





①
$$F_{Net} = F_N - F_g$$
$$\frac{mv^2}{r} = F_N - mg$$
$$\frac{mv^2}{r} + mg = F_N$$
$$m\left(\frac{14^2}{2.8} + 9.8\right) = F_N$$
Eq 1

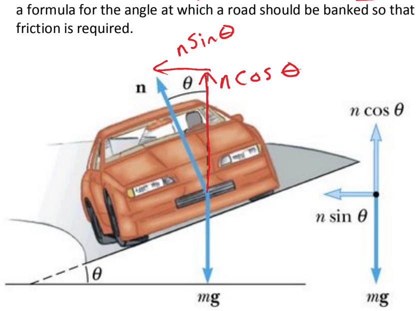
③
$$F_{Net} = F_N + F_g$$
$$\frac{mv^2}{r} = F_N + mg$$
$$m\left(\frac{v^2}{r} - g\right) = F_N$$
$$m\left(\frac{v^2}{2.8} - 9.8\right) = F_N$$
Eq 3

Sub Eq 1 → Eq 3
$$\cancel{m}\left(\frac{14^2}{2.8} + 9.8\right) = \cancel{m}\left(\frac{v^2}{2.8} - 9.8\right)$$
$$\sqrt{2.8\left(\frac{14^2}{2.8} + 19.6\right)} = v$$
$$\boxed{15.8\,m/s = v_{Top}}$$

Driving on a Frictionless Banked Corner.

For a car travelling with speed v around a curve of radius r, determine

Driving on a Frictionless Banked Corner.
For a car travelling with speed **v** around a curve of radius **r**, determine a formula for the angle at which a road should be banked so that no friction is required.



Horizontal

$F_{Net} = n\sin\theta$

$ma_c = n\sin\theta$

$\frac{mv^2}{r} = n\sin\theta$ (Eq 1)

Vertical (a=0)

$\cancel{F_{Net}} = n\cos\theta - mg$

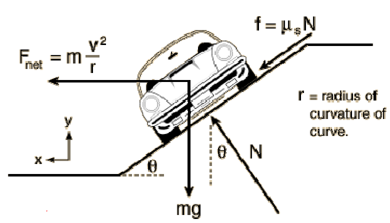
$0 = n\cos\theta - mg$

$mg = n\cos\theta$ (Eq 2)

$\frac{Eq\ 1}{Eq\ 2}$ $\frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g} = \frac{\cancel{n}\sin\theta}{\cancel{n}\cos\theta}$

$\frac{v^2}{rg} = \tan\theta$

$\tan^{-1}\left[\frac{v^2}{rg}\right] = \theta$



Force equations at maximum speed v, at threshold of sliding up incline.

$\Sigma F_x = m\frac{v^2}{r} = N\sin\theta + \mu_s N\cos\theta$

$\Sigma F_y = 0 = N\cos\theta - \mu_s N\sin\theta - mg$

Solving this pair of equations for the maximum speed v gives:

$$v_{max} = \sqrt{\frac{rg(\sin\theta + \mu_s\cos\theta)}{\cos\theta - \mu_s\sin\theta}}$$

The limiting cases are:

$v_{max} = \sqrt{rg\tan\theta}$ Frictionless case

$v_{max} = \sqrt{rg\mu_s}$ Flat roadway