Momentum in 1D - Physics 11

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Momentum of a particle is the product of its mass and its velocity. As velocity is a vector quantity, so momentum itself is also a vector (i.e. it has both magnitude and direction) It has the same direction as the direction of the object in motion.

Momentum

Symbol: $\vec{p}$

SI Unit: kilogram meter per second (kg x m /s direction)

(or Newton Seconds N·s)

$$\boxed{\vec{p} = m\vec{v}}$$

A heavy object moving slowly can have the same momentum as a light object moving quickly.

Ex 1: Compare the momentum of a 0.1 kg bullet moving at 1000 m/s and a 60 kg student moving at 1.7 m/s (6 km/h)

$p_{bullet} = (0.1)(1000)$
$= \underline{100\ kg\,m/s}$

$p_{student} = (60)(1.7)$
$= \underline{102\ kg\,m/s}$

Ex 2: How fast does a 200g baseball need to be moving to have the equivalent momentum as a 5 kg bowling ball moving 8 m/s down a bowling lane?

$$p_1 = p_2$$
$$(0.2)(v) = (5)(8)$$
$$v = \frac{(5)(8)}{0.2}$$
$$\underline{\underline{v = 200\ m/s}}$$

To change the velocity of an object we need to apply a force to it for a period of time. This will give the object an acceleration, but that acceleration will be dependent on the force applied and the mass of the object (F=ma).

What we find is that heavy objects are harder to stop or move then lighter ones.

Impulse

Impulse is the force exerted on an object over a period of time.

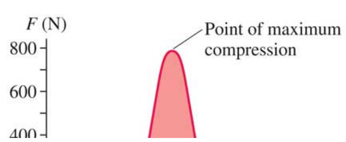
$$Imp = F \cdot t$$

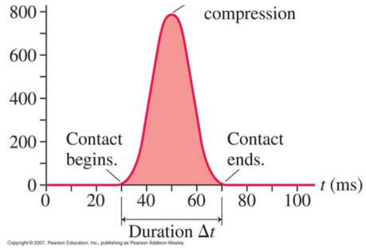
Units
N·s
kg m/s

$Imp = p_f - p_i$

Graphically the impulsive force may look like the graph below;

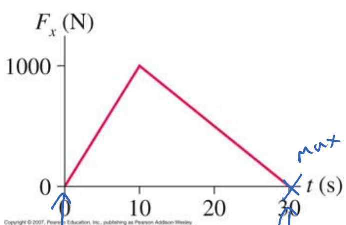
Impulse = Area under a Force vs. time Graph





Impulse = ' Gray

Far in space, where gravity is negligible, a 425 kg rocket traveling at 75 m/s fires its engine. The figure shows the thrust force as a function of time. The mass lost by the rocket during these 30.0 s is negligible.
a. What impulse does the engine impart to the rocket?
b. At what time does the rocket reach it maximum speed? What is the maximum speed?



max Speed

V_i V_f

$$\text{Impulse} = \text{Area under Curve}$$
$$= \tfrac{1}{2} bh$$
$$= \tfrac{1}{2}(30)(1000)$$
$$= 15000 \text{ N}\cdot\text{s}$$

$V_i = 75$ m/s
$m = 425$ kg

$$\text{Impulse} = \Delta P$$
$$15000 = mV_f - mV_i$$
$$\frac{15000 + mV_i}{m} = V_f$$
$$V_f = \frac{15000 + (450)(75)}{450} = 108 \text{ m/s}$$

From the problem above:

$$Imp = \Delta p$$
$$Imp = m(\overrightarrow{v_f} - \overrightarrow{v_i})$$

Combining the equation of impulse before with the one above we get:

$$\vec{F} \cdot t = m(\overrightarrow{v_f} - \overrightarrow{v_i})$$

Ex 1: a) What impulse is needed to make a 60 kg person moving at 5 m/s to end up moving at 2 m/s?

$Imp = F \cdot t$
$Imp = m(V_f - V_i)$

$$\text{Impulse} = m(V_f - V_i)$$
$$= 60(2-5)$$
$$= 60(-3)$$
$$= -180 \text{ kg m/s}$$

b) If the impulse used a force of -20N. How long would this force need to be incontact with the person?

$Impulse = F \cdot t$

$-180 = -20 \cdot t$

$\frac{-180}{-20} = t$

$\boxed{9s = t}$

c) What force would need to be used to change this persons velocity in 0.5s?

$Impulse = F \cdot t$

$-180 = F(0.5)$

$\frac{-180}{0.5} = F$

$\boxed{-360N = F}$

2. An impulse of -25 kgm/s is imparted upon a 2 kg object moving at 3m/s.
   what is its new velocity?

$Imp = m(V_f - V_i)$

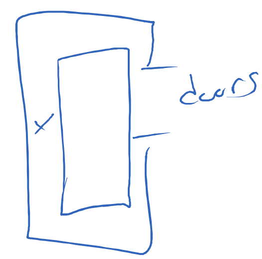
$\frac{-25}{2} = \frac{2(V_f - 3)}{2}$

$-12.5 = V_f - 3$

$+3 \qquad +3$

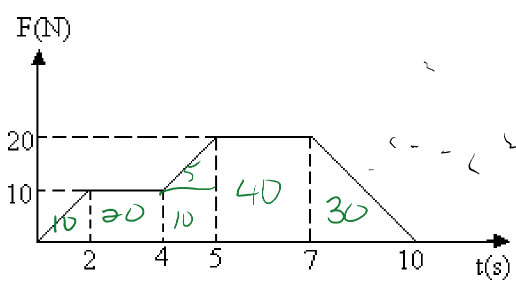
$\boxed{-9.5 m/s = V_f}$

- Burning Bus (4)
- Sweater Fire (10)
- Bears (short) (1)



Impulse Graphs - The area under a force vs. time graph give us the impulse.

A) A 20 kg object experiences an impulse from the graph above, from 10 m/s what is the new velocity?



$Imp = F \cdot t$

$= \text{Area under the curve}$

$Imp = 115 N \cdot S$

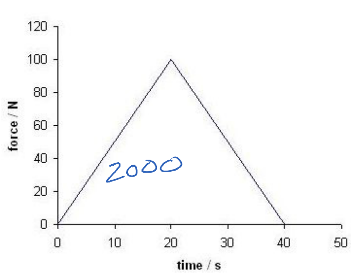
$115 = m(V_f - V_i)$

$115 = 20(V_f - 10)$

$\frac{115}{20} + 10 = V_f$

$V_f = 15.75 m/s$

What is the final velocity of the 5 kg block, initially at rest, after it has had an impulse imparted(applied) on it. Use the graph to help you.



$Imp = \text{Area under Curve}$

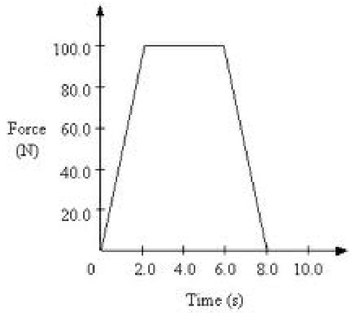
$= 2000 N \cdot S$

$Imp = m(V_f - V_i)$

$\frac{2000}{5} = \frac{5(V_f)}{5}$

$\boxed{400 m/s = V_f}$

If the mass of the object is 3.0 kg, what is its final velocity over the 8.0 s time period if it starts from rest?



Conservation of Momentum

Closed System: Does not exchange any matter with its surroundings and no external forces act on the system.

Conservation of Momentum tells us that in a closed system the total initial momentum equals the total final momentum. In other words no momentum is lost.

$$\overrightarrow{p_i} = \overrightarrow{p_f}$$

In this section we will explore what happens to the momentum of a system in three types of collisions.

Collisions (Non-stick)	totally Elastic
Collisions (Stick)	inelastic
Explosions (break apart)	inelastic

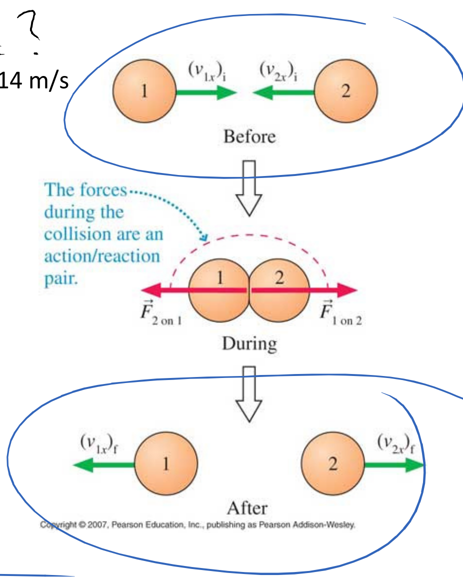
Below is a totally elastic collisions. Find the Velocity of Object 1.

Given:

$M_1 = 4$ kg $\quad V_{1i} = 8$ m/s $\quad V_{1f} = ?$

$M_2 = 3$ kg $\quad V_{2i} = -8$ m/s $\quad V_{2f} = 14$ m/s

initial



final

$P_i = m_1 V_{1i} + m_2 V_{2i}$

$= (4)(8) + (3)(-8)$

$P_i = 8$ kg m/s

$P_f = m_1 V_{1f} + m_2 V_{2f}$

$= (4)(V_{1f}) + (3)(14)$

$P_f = 4V_{1f} + 42$

$P_i = P_f$

$8 = 4V_{1f} + 42$

$-42 \qquad -42$

$-34 = 4V_{1f}$

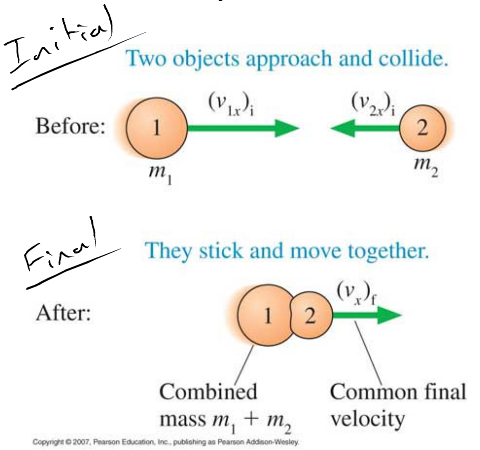
$\boxed{-8.5 \text{ m/s} = V_{1f}}$

$-42 \qquad -42$

$\dfrac{-34}{4} = \dfrac{4V_{1f}}{4}$

$\boxed{-8.5\,m/s = V_{1f}}$

Below is a inelastic collision where the two objects collide and stick together. Find the mass of Object 2.



M_1 = 5.0kg
V_{1i} = 6.0m/s
V_{2i} = -4.0m/s
V_f = 1.5m/s

$P_i = m_1 V_{1i} + m_2 V_{2i}$

$P_i = 5(6) + m_2(-4)$

$P_i = 30 - 4m_2$

$P_f = (m_1 + m_2) V_f$

$= (5 + m_2)(1.5)$

$P_f = 7.5 + 1.5m_2$

$P_i = P_f$

$30 - 4m_2 = 7.5 + 1.5m_2$

$30 - 7.5 = 1.5m_2 + 4m_2$

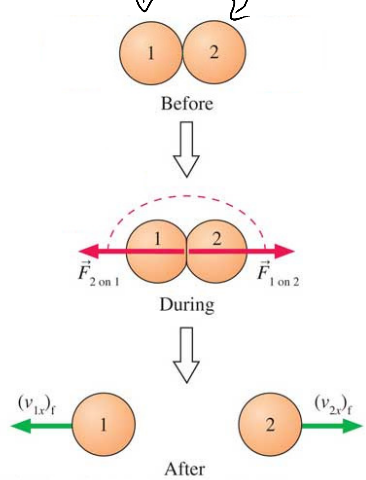
$\dfrac{22.5}{5.5} = \dfrac{5.5m_2}{5.5}$

$\boxed{m_2 = 4.1\,kg}$

Below is an explosion. Find the relative velocities of the masses M_1=12kg and M_2=9kg. If the initial momentum of the system is 0.

12kg Tyler

9kg Seb



$P_i = 0$

$P_f = m_1 V_{1f} + m_2 V_{2f}$

$P_f = 12V_{1f} + 9V_{2f}$

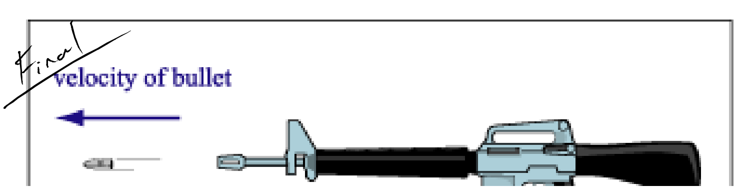
$P_i = P_f$

$0 = 12V_{1f} + 9V_{2f}$

$\dfrac{-9V_{2f}}{-9} = \dfrac{12V_{1f}}{-9}$

$\boxed{V_{2f} = -\frac{4}{3}V_{1f}}$

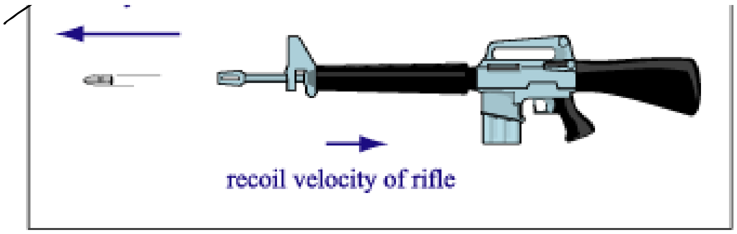
Find the recoil velocity of the ~~10~~ 7kg gun if it fires a 9.5g bullet out at a velocity of 900m/s. (Assume that before the gun is fired it is not moving)



$P_i = m_i v_i$

$P_i = (7 + 0.0095)(0)$

$P_i = 0\,kg\,m/s$



$P_i = (7 + 0.0095)(0)$

$P_i = 0\ kg\ m/s$

$P_f = (0.0095)(900) + (7)V_{recoil}$

$P_i = P_f$

$0 = -8.55 + 7V_{recoil}$

$+8.55 \quad +8.55$

$\frac{8.55}{7} = \frac{7V_{recoil}}{7}$

$\boxed{1.22\ m/s = V_{recoil}}$

What if the 67kg person holding the gun puts their full weight behind it?

Same as before but add more mass to the gun i.e. 10→77

$0 = (0.0095)(900) + 77V_f$

$V_f = -\frac{(0.0095)(900)}{77}$

$V_f = -0.11\ m/s$

The ballistic pendulum was used to measure the speeds of bullets before electronic timing devises were developed. The version shown in the figure consists of a large block of wood of mass M = 5.4 kg, hanging from two long cords. A bullet of mass m = 9.5 g is fired into the block, coming quickly to rest. The block+bullet then swing upward, their center of mass rising a vertical distance h = 6.3 cm, with this information they were able to find the final velocity of the bullet pendulum system to be 1.11m/s. What was the initial velocity of the bullet.