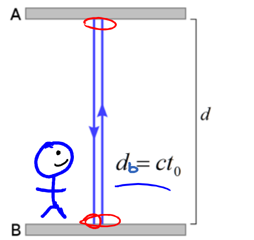
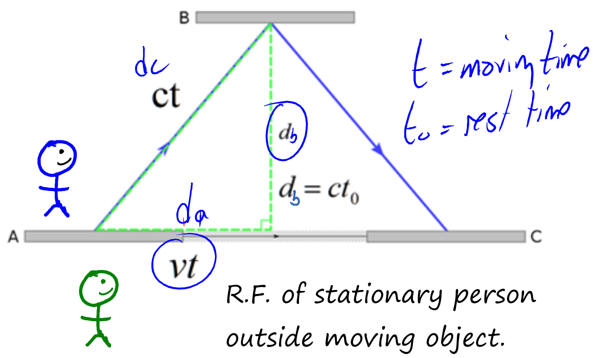
Time Dilation

September 8, 2015 1:28 PM

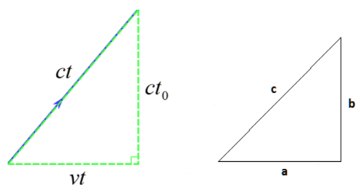
Below is a situation where you have a blue person inside a fast moving object (velocity, **v**) and a stationary green person outside watching the object go flying past. A ray of light (moving at speed of light, **c**) is sent from the floor B up to the ceiling, A , reflect and come back down to B.



R.F. of person inside moving object.



R.F. of stationary person outside moving object.



$\Rightarrow \quad a^2 \quad + \quad b^2 \quad = \quad c^2$ (pythogorean theorem)

$\Rightarrow (vt)^2 + (ct_0)^2 = (ct)^2$ (substitute values)

$\Rightarrow v^2t^2 + c^2t_0{}^2 = c^2t^2$ (expand)

$\Rightarrow \frac{v^2t^2}{c^2} + \frac{\cancel{c^2}t_0{}^2}{\cancel{c^2}} = \frac{\cancel{c^2}t^2}{\cancel{c^2}}$ (divide by c^2)

$\Rightarrow \frac{v^2t^2}{c^2} + \underline{t_0}{}^2 = t^2$ (isolate t_0)

$\Rightarrow t_0{}^2 = t^2 - \frac{v^2t^2}{c^2}$ (factor out t^2)

$\Rightarrow t_0{}^2 = t^2\left(1 - \frac{v^2}{c^2}\right)$ (isolate t) $\qquad \frac{t_0{}^2}{1-\frac{v^2}{c^2}} = t^2$

$\Rightarrow \sqrt{t^2} = \sqrt{\frac{t_0{}^2}{\left(1-\frac{v^2}{c^2}\right)}}$ (square root everything)

$\Rightarrow t = \frac{t_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = \frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}t_0$

Let $\boxed{\beta = \frac{v}{c}}$ and $\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}}}$ $\quad v =$ Speed of the refrence frame

$\beta < 1$

$\gamma > 1$

the equation becomes...

$$\boxed{t = \gamma t_0}$$

t: the time dilation

t_0: the proper time or rest time $\qquad c = 3.00 \times 10^8 \, m/s$

Find γ for an object traveling at $\underline{\underline{0.9c}} = (0.9)(3.00 \times 10^8 \, m/s)$

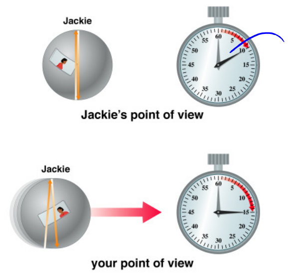
$\beta = \frac{v}{c} = \frac{0.9\cancel{c}}{\cancel{c}} = 0.9$

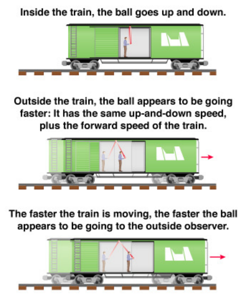
$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{0.19}} = 2.294$

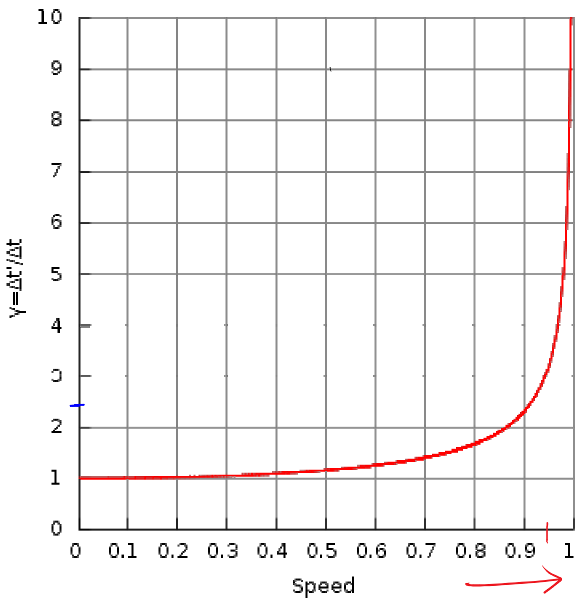
Since v is always less than c, gamma (γ) will always be greater than 1, that is $t_o < t$

t_o is referred to as proper time (the time interval between two events as measured by an observer who sees the events occur at the same place). That is proper time is always the time measured with a single clock at rest in that frame.

According to a **stationary observer**, a **moving** clock runs slower than an identical **stationary** clock. This effect is known as time dilation.

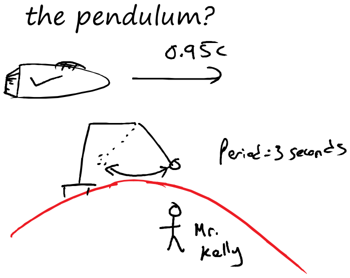






Example: What is the Period of the Pendulum?

The period of a pendulum is measured to be 3.0 seconds in the inertial frame of the pendulum. What is the period of the pendulum when measured by an observer moving at a speed of 0.95c with respect to the pendulum?



$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$v = 0.95c$

$v^2 = 0.95^2 c^2$

$$= \frac{1}{\sqrt{1-\frac{0.95^2 \cancel{c^2}}{\cancel{c^2}}}}$$

$$\gamma = 3.2$$

$t_0 = 3$

$t = \gamma t_0$

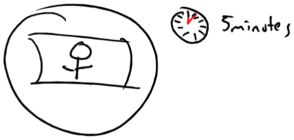
$= (3.2)(3)$

$t = 9.6$ seconds

Example: You see your friend fly by you in a spaceship moving at a rate of 0.95c. According to your watch, you observe your friend holding their breath for 5 minutes. How long did they actually hold their breath?

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad v = 0.95c$$

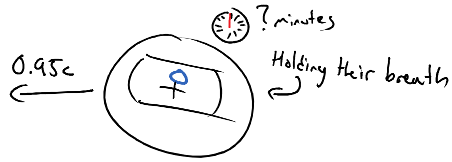
$$\gamma = 3.2 \qquad \text{Same as above}$$



$$t = \gamma t_0$$

$$\frac{5}{3.2} = \frac{3.2\, t_0}{3.2}$$

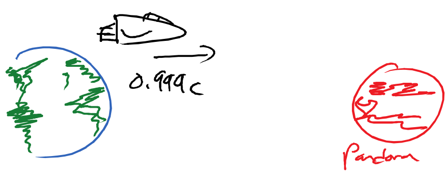
$$t_0 = \underline{\underline{1\text{ min } 34 \text{ seconds}}}$$



Example: 17 year-old Chris is flying to Pandora to see his beautiful alien girlfriend. He flies at .999c for 100 years (according to Earthlings.) How old is Chris when he arrives?

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.999^2}}$$

$$= \underline{\underline{22.4}}$$



$$t = \gamma t_0$$

$$\frac{100}{22.4} = \frac{22.4\, t_0}{22.4}$$

$$4.5 \text{ years} = t_0$$

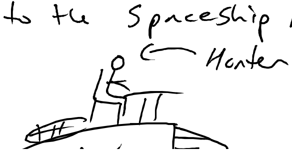
He is $\underline{\underline{21.5 \text{ years old}}}$

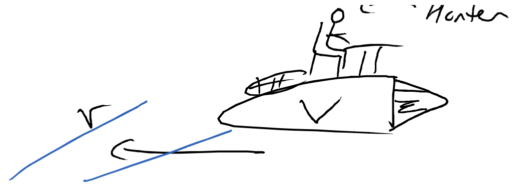
One lightyear= v*t=3e8m/s*365day*24h/day*60min/h*60sec/min=9.461x10^15 m=9.461x10^12

$\boxed{9{,}461{,}000{,}000{,}000 \text{ km}}$

On the Parkland spaceship, a physics test is 80 minutes long, just like the Earth Parkland physics class. How fast does the ship need to travel in order for someone in Earth's physics class to ~~be~~ appear to have 200 minutes ~~long~~? (Answer in c and m/s) (1 c = 3x10^8 m/s)

to write the test according to the Spaceship kids.





$$t = \gamma t_0$$

$$\frac{200}{80} = \frac{\gamma 80}{80}$$

$$2.5 = \gamma$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\left(\frac{1}{\gamma}\right)^2 = \left(\sqrt{1 - \frac{v^2}{c^2}}\right)^2$$

$$\frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2}$$

$$\sqrt{\frac{v^2}{c^2}} = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$v = \sqrt{1 - \frac{1}{\gamma^2}}\; c$$

$$\underline{\underline{v = 0.917c}}$$