t_0 : Proper Time
t : movin Time

Length Contraction

September 8, 2015 1:28 PM

Length contraction: If an observer at rest with respect to an object measures its length to be L_O, an observer moving with a relative speed v with respect to the object will find it to be shorter than its rest length by the factor $1/\gamma$. Only the dimension that is in parallel with the direction of travel will change.

If the triangle 0.95c

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

Let $\beta = \frac{v}{c}$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

the equation becomes...

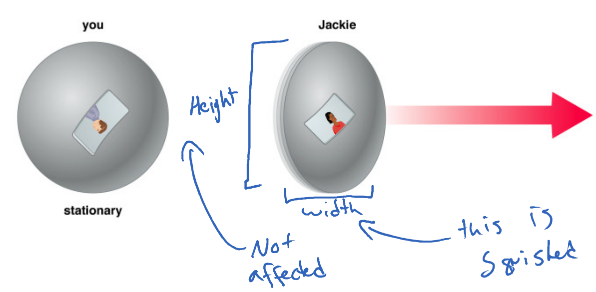
$$\boxed{L = \frac{L_0}{\gamma}}$$

L_0 = Rest Length
L = Moving Length

L: the contracted length

L_0: the proper length or rest length

The **proper length** of an object is defined as the length of the object measured in the reference frame in which the object is at rest



Example: The Contraction of a Spaceship

Proper Length

A spaceship is measured to be 100 meters long while it is at rest with respect to an observer. If this spaceship now flies by the observer with a speed of 0.99c, what length will the observer find for the spaceship?
If the ship moves past the observer with a speed of 0.01c,
what length will the observer measure?

$L_0 = 100\text{m}$

$v = 0.99c$

$\gamma = \frac{1}{\quad}$

$v = 0.01c$

$\gamma = \frac{1}{\quad}$

$L_0 = 100m$

$v = 0.99c$

$$\gamma = \frac{1}{\sqrt{1-0.99^2}}$$

$$\gamma = 7.1$$

$$L = \frac{L_0}{\gamma}$$

$$L = \frac{100}{7.1}$$

$$\underline{\underline{L = 14m}}$$

$v = 0.01c$

$$\gamma = \frac{1}{\sqrt{1-0.01^2}}$$

$$\gamma = 1.0\;0005$$

$$L = \frac{100}{1.00005}$$

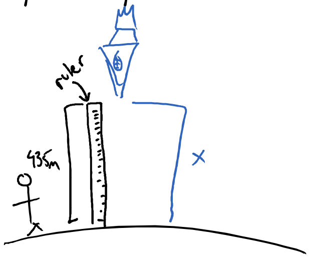
$$\underline{\underline{L = 99.995m}}$$

Example: How High is the Spaceship?

An observer on Earth sees a spaceship at an altitude of 435 meters moving downward towards the Earth with a speed of 0.97c. What is the altitude of the spaceship as measured by an observer in the spaceship?

$$\gamma = \frac{1}{\sqrt{1-0.97^2}}$$

the altitude of the spaceship as measured by an observer in the spaceship?



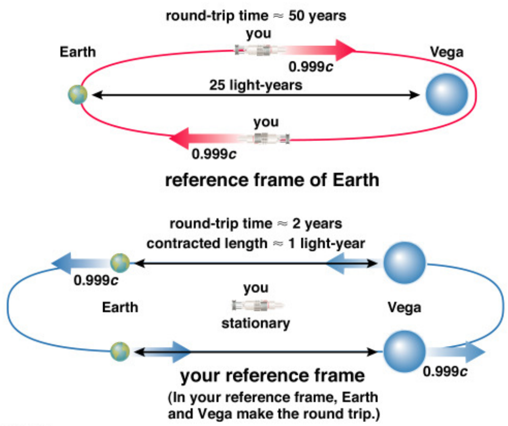
$L_0 = 435 m$

$L = \frac{435}{4.11}$

$\underline{L = 106 m}$

$\gamma = \frac{1}{\sqrt{1-0.97^2}}$

$\gamma = 4.11$

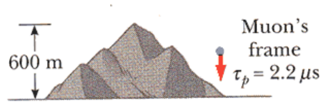


Earlier we mentioned **proper time** is the time measured in the moving object, not the stationary person. But **proper length** is measured when the object is at rest. The person moving sees objects shorter just as the stationary person sees the moving object shorter.

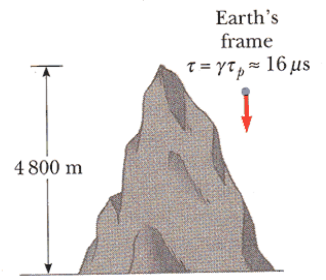
Muons are sub-atomic particles that have a life time of 2.2μs (2.2 millionth of a second). They are created by high energy cosmic rays (hydrogen atoms minus the electron, so just a proton) that smash into our atmosphere (35 km above sea level), hitting Nitrogen or Oxygen molecules. The collision creates Pions that decay (very quickly) into Muons. These Muons are travelling close to the speed of light (0.995c). We can record many Muons at sea level. With such a short lifetime, they should only go d=vt= $2.985*10^8$ m/s x 0.0000022 s = 656.7 meters. Yet we see them at sea level, why? $\gamma = \frac{1}{\sqrt{1-.995^2}} = 10.01$

This is because of special relativity. Two different things are happening in each frame of reference.

For us on the Earth, the Muons lifespan has increased due to their clocks running more slowly at high velocities, BUT for the Muons, they still only live for 2.2μs but 35 km shrinks down to 600 meter due to length contractions. Everybody wins!



(a)



(b)

Formulas

$\beta = \frac{v}{c}$ $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ $c = 3.00 \times 10^8$ m/s

$t = \gamma t_0$ t: moving time
t_0: proper time

$L = \frac{L_0}{\gamma}$ L: moving length
L_0: proper length