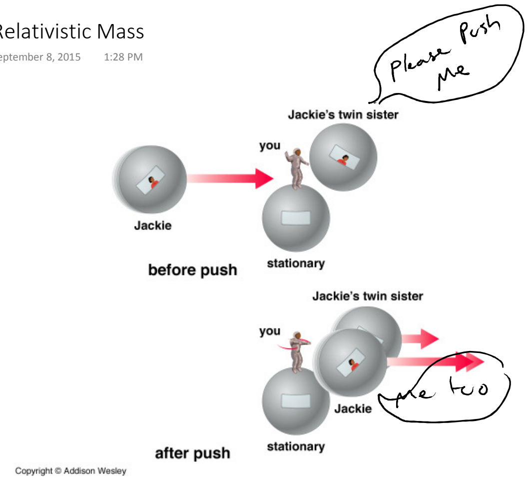
Relativistic Mass

September 8, 2015 1:28 PM



Imagine that Jackie's twin sister is in her spacecraft and You are floating out in space right next to the craft ready to give it a push. Jackie's ship comes along at a relative speed, v. Now both Jackie and Jackie's twin sister are identical in mass and so are their ships (at rest anyway). You give them both an identical push at the moment that Jackie passes you. That means you applied an identical force. But, Jackie's clock is running slow as seen by you and Jackie's twin sister. Hence your push on Jackie's ship happens over a shorter time than the push on Jackie's twin sister's ship. Therefore your equal pushes produce different accelerations on Jackie and Jackie's twin sister's ships. Jackie's ship accelerates less because it felt the force for a shorter time. The only way this can be is if you measure Jackie's mass to be greater than Jackie's twin sister's.

$F = ma$ or $a \propto (1/m)$

From your point of view, objects moving past you have a greater mass than they have at rest. The faster an object is moving the greater the increase in mass. As you increase the speed toward the speed of light the mass becomes infinte. Thus it would take infinte force or infinite energy to accelerate any mass to the speed of light. This is why it was stated earlier that no material object may move at the speed of light.

The observed mass of an object increases with speed according to the relation:

$$M = \frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Let $\beta = \frac{v}{c}$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

the equation becomes...

$$\boxed{M = \gamma M_0}$$

M: the relativistic mass

M_0: the rest mass

Example: Mass of an Electron

An electron, which has a rest mass of 9.11×10^{-31} kg, has been accelerated to a velocity of 0.995c in a particle accelerator. What is its relativistic mass?

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.995^2}}$$

$$= \underline{\underline{10.0}}$$

$$M = \gamma M_0$$
$$= 10(9.11 \times 10^{-31})$$
$$\underline{\underline{M = 9.11 \times 10^{-30}\ kg}}$$

For the electron to have a $M = 1\,kg$

$$\frac{M}{M_0} = \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\left(\frac{M_0}{M}\right)^2 = \sqrt{1-\frac{v^2}{c^2}}^{\,2}$$

$$\sqrt{1-\left(\frac{M_0}{M}\right)^2} = \sqrt{\frac{v^2}{c^2}}$$

$$v = \sqrt{1-\left(\frac{M_0}{M}\right)^2}\, c$$

$=$

Too close to c

$$v = \sqrt{1-10^{-62}}\, c$$
$$v = 1-10^{-31}\, c$$
$$v = \underbrace{0.99999 \ldots\ldots 9}_{31}\, c \sim c$$

~~Chris~~ Robin is moving at a velocity of 2.9954 x 10^8 m/s and his mass is 450 kg. What is his rest mass?

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1-\left(\frac{2.9954 \times 10^8}{3.00 \times 10^8}\right)^2}}$$

$$\gamma = 18.1$$

$$M = \gamma M_0$$

$$\frac{450}{18.1} = \frac{18.1 M_0}{18.1}$$

$$\underline{\underline{25\,kg = M_0}}$$

Harder Question

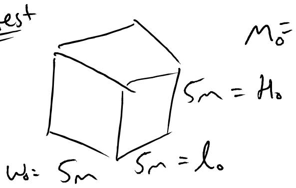
A box with rest dimensions of 5m travels @ 0.997c and has a rest mass of 28 kg. What is the density of the box measured by a stationary observer?

@ 0.997c

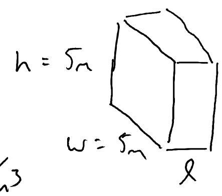
28 kg. What is the density of ... observer?

@ rest

$m_0 = 28\,kg$



@ 0.997c →



$m = \gamma m_0$

$l = \frac{l_0}{\gamma}$

$$\rho_0 = \frac{\text{Mass}}{\text{Volume}} = \frac{28}{5\times5\times5} = 0.224\,kg/m^3$$

density

$$\gamma = \frac{1}{\sqrt{1-0.997^2}}$$

$$\underline{\underline{\gamma = 12.9}}$$

$$\rho = \frac{m}{V} = \frac{m}{H_0 \times W_0 \times l}$$

$$\rho = \frac{\gamma m_0}{H_0 \times W_0 \times \frac{l_0}{\gamma}} \cancel{\frac{\gamma}{\gamma}}$$

$$\rho = \gamma^2 \frac{m_0}{H_0 \times W_0 \times l_0}$$

$$\rho = \gamma^2 \rho_0$$

$$= (12.9)^2 (0.224)$$

$$\boxed{\rho = 37.4\,kg/m^3}$$