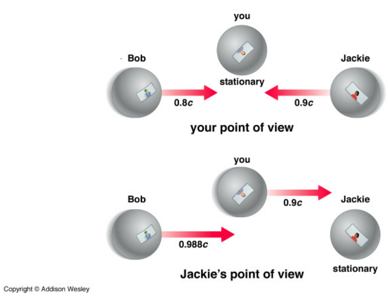
Homework Q1-7 wrkst #2

In Newtonian mechanics, we learned vector addition. So if a boat is motoring downstream, its velocity observed by a stationary person would be just the velocity of the boat plus the current of the river. Because objects cannot go faster than the speed of light, we need a new type of way to add or subtract velocities nearing the speed of light (at least 10%). This is called the Lorentz velocity transformation, which is the relativistic counterpart of the Galilean velocity transformation. A spaceship moving at 0.7c fires a rocket at 0.5c does not mean the rocket is going 0.7+0.5= 1.2c according to someone standing stationary watching it.

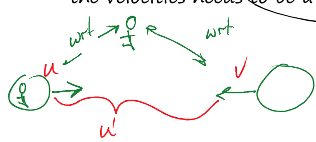


Lorentz velocity transformation for S→S' (S is at rest, S' is moving)

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

u': velocity of moving object w.r.t other moving object
u: velocity of object w.r.t stationary 3rd party observer
v: velocity of other object w.r.t stationary 3rd party observer

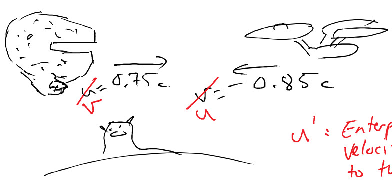
If the velocities are in opposite directions, then one of the velocities needs to be a negative number!!!!!!!!!!!!



Example: Relative Velocity of Spaceships

Two spaceships Millennium Falcon and Enterprise are moving in **opposite** directions. An observer on the Earth measures the speed of the Millennium Falcon to be 0.75c and the speed of the Enterprise to be 0.85c. Find the velocity of the Enterprise with respect to the Millennium Falcon moving towards each other

a)



$u' = \frac{u - v}{1 - \frac{uv}{c^2}} =$

u' : Enterprises velocity relative to the falcon

$$u' = \frac{-0.85c - 0.75c}{1 - \frac{(-0.85\cancel{c})(0.75\cancel{c})}{\cancel{c^2}}} = \frac{-1.6c}{1 + (0.85)(0.75)} = \frac{-1.6c}{1.6375} = \underline{-0.977c}$$

$v = 0.977c$

b) If the captain of the Enterprise measures the Falcon at 10 m when they pass, what is the Millennium Falcon's rest length?

$l = 10m$
$l_0 =$
$\gamma = 4.7$

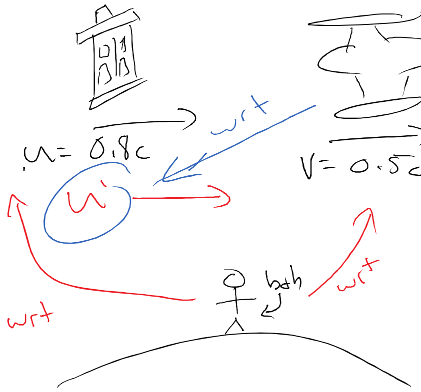
$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} =$

$\gamma = 4.7$

$$l = \frac{l_0}{\gamma} \Rightarrow l_0 = l\gamma = (10)(4.7)$$

$$\underline{l_0 = 47m}$$

The "tardis" is travelling at 0.8c in the same direction as "Serenity" which is going at 0.5c. What is the relative velocity that Serenity approaches someone in the tardis?



$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{0.8c - 0.5c}{1 - \frac{(0.8)(0.5)\cancel{c^2}}{\cancel{c^2}}}$$

$$= \frac{0.3c}{0.6}$$

$$u' = \underline{\underline{0.5c}}$$

The tardis looks like it is going at 0.5c towards someone on Serenity.

$$u' = \frac{u - v}{\frac{c^2 - uv}{c^2}} \Rightarrow u' = \frac{(u-v)c^2}{c^2 - uv} = u'(c^2 - uv) = (u-v)c^2$$

$$= u'c^2 - uvu' = c^2u - c^2v$$

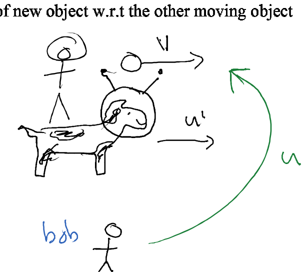
$$u'c^2 + c^2v = c^2u + uvu'$$

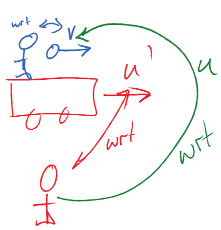
$$c^2(u' + v) = u(c^2 + u'v) \qquad \boxed{u = \frac{c^2(u'+v)}{c^2 + u'v}}$$

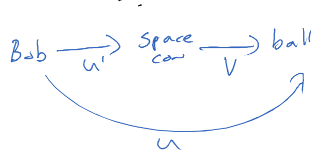
Lorentz velocity transformation for S'→S (S' is moving, S is at rest)

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

u: apparent velocity of new moving object w.r.t stationary 3rd party observer
u': velocity of an object w.r.t stationary 3rd party observer
v: velocity of new object w.r.t the other moving object

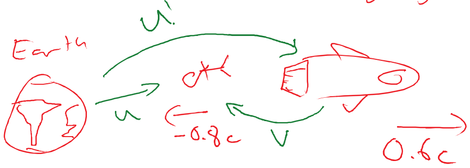






We are on the spaceship Bob travelling at 0.6c when we fire Grace out of the ship back towards earth at 0.8c w.r.t. ~~us~~ Space Ship.
How fast is she going w.r.t. earth?
u'

fire Grace out of 'the ship' back towards earth at 0.8c w.r.t. ~~us~~. Space Ship
How fast is she going w.r.t. earth?

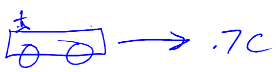


$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{0.6c - 0.8c}{1 + \frac{(0.6c)(-0.8c)}{c^2}}$$

$$= \frac{-0.2c}{0.52}$$

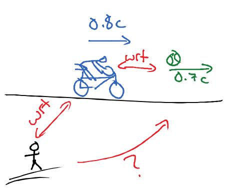
$$= \underline{\underline{-0.38c}}$$

$v = -0.8c$
$u' = 0.6c$



$$u = \frac{.7c + .3c}{1 + \frac{\cancel{.7c}\,.3\cancel{c}}{\cancel{c^2}}} = \frac{1.5c}{1.56} = 0.962c$$

Example: The Speeding Motorcycle
Imagine a motorcycle rider moving with a speed of 0.8c past a stationary observer. If the rider tosses a ball in the forward direction with a speed of 0.7c relative to himself, what is the speed of the ball as seen by the stationary observer?



$u' = 0.8c$
$v = 0.7c$
$u = ?$

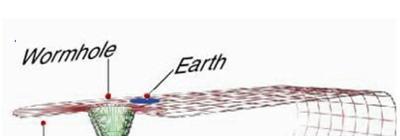
$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

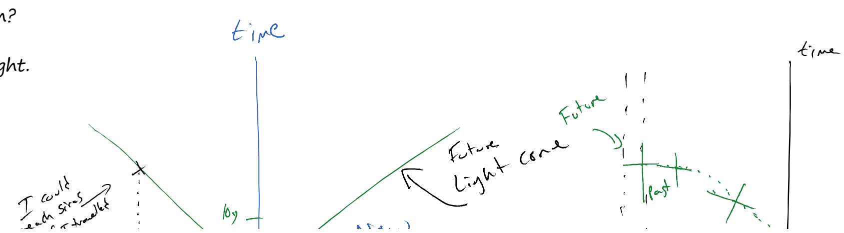
$$= \underline{\underline{0.96c}}$$

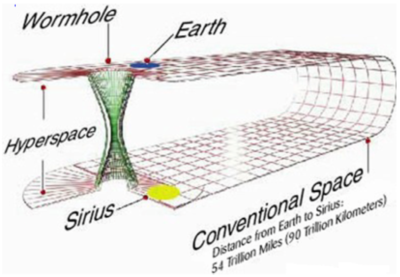
Tangent

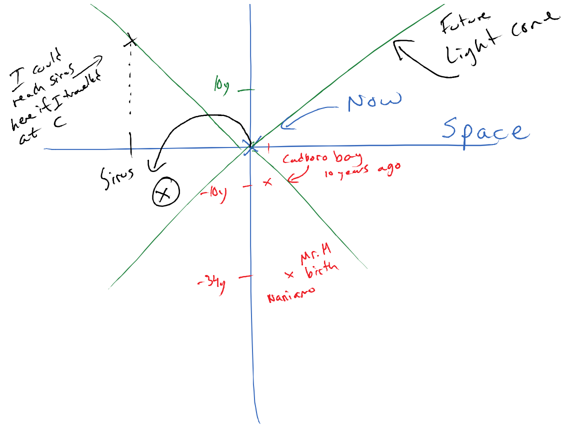
Part B: Suppose that the motorcyclist moving with a speed of 0.8c turns on his headlight. How fast does the beam move away from him? What would the stationary observer measure for the speed of the beam?

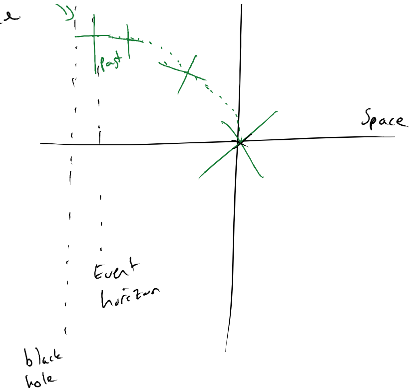
In theory, this is the only way to go faster than the speed of light.



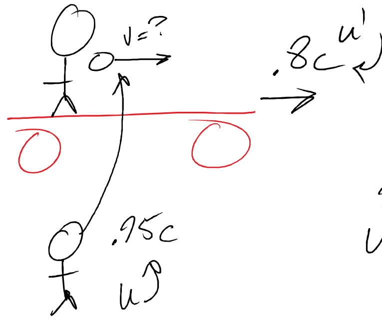








ex)



$$u\left(1+\frac{u'v}{c^2}\right)=u'+v$$

$$u+\frac{uu'v}{c^2}=u'+v$$

$$\frac{u\cdot u'\cdot v}{c^2}-v=u'-u$$

$$v\left(\frac{u\cdot u'}{c^2}-1\right)=u'-u$$

$$u=\frac{u'+v}{1+\frac{u'v}{c^2}}$$

$$v=\frac{u'-u}{\frac{u\cdot u'}{c^2}-1}=\frac{.8-.95}{\frac{.8\cancel{c}\cdot .95\cancel{c}}{\cancel{c^2}}-1}=\frac{-0.15}{-0.24}=.625c$$

$$u=\frac{.8+.625}{1+.8\cdot .625}=\frac{1.425}{1.5}=0.95$$

$$\boxed{u = \frac{c^2(u'+v)}{c^2+u'v}}$$

$M = \gamma M_0$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

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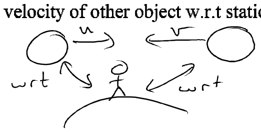

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

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