Answers to Worksheet

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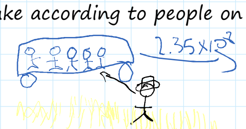
Ans:
1. 20.1 mins
2. 2.43 mins
3. 24 sec
4. 1325 m
5. 154.1 m
6. 0.934c, 2.79×10^8 m/s
7. a) 4.8 m, b) 4.8 m, c) Yes
8. $v = 0.8c$, $t = 5$ years

1. A very long train full of people travels at 2.35×10^8 m/s past a farmer working in a field. According to the farmer's watch, it takes 12.5min to shoe a horse. How long does the shoeing take according to people on the train?



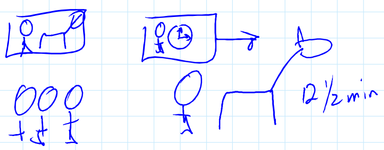
$\beta = \frac{v}{c} = \frac{2.35 \times 10^8}{3 \times 10^8} = .78$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-.78^2}} = 1.6$ $\quad \boxed{t = \gamma t_0} \quad t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} t_0$

$t = ?$

t_0 (rest time) $= 12.5$

$t = 1.6 \times 12.5 = \underline{20.1\ \text{min}}$



2. A second train passes the same farm traveling at0.72c The people on board notice that it takes 3.5 min for the farmers son to milk a cow. How long does the son say it takes?

$\beta = \frac{.72c}{1c} = .72 \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.44$

$t = 3.5\,m \quad t = \gamma t_0 \quad \boxed{t_0 = \frac{t}{\gamma}} = \frac{3.5}{1.44} = \underline{2.43\,min}$

$t_0 = ?$

$\underline{t_0 = 2.43\ min}$

3. Two very long trains pass each other head on with a relative speed of 0.97c. Bob, the driver of the first train notices that it takes 5.8s on his watch to pass the entire second train. How long do people on the second train say it takes for Bob to go by their train?

$\beta = .97 \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} = 4.11 \quad t = \quad t_0 = 5.8\,s \quad t = 4.11 \times 5.8 \underline{\approx 24\ sec}$

4. A train that has a proper length of 2000m is traveling through a station at a speed of 0.75c. How long does it appear to be to an observer standing in the station?

$\beta = .75 \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.51 \quad L = \frac{L_0}{\gamma} = \frac{2000}{1.51} = \underline{1325\,m}$

$L_0 = 2000\,m$

$\boxed{L = 1325\ m}$

5. If a second train heads through the station at 0.97c and appears to be 37.5m long to an observer standing in the station, what is the trains proper length? $\beta = .97 \quad \gamma = 4.11 \quad L = \frac{L_0}{\gamma}$

$L = 37.5 \quad L_0 = \gamma L = 4.11 \times 37.5 = 154.1\ m$

6. At what speed will a 7.0m long car appear to be only 2.5m long to a person standing on the road? Give your answer as both m/s and a C

6. At what speed will a 7.0m long car appear to be only 2.5m long to a person standing on the road? Give your answer as both m/s and a C value.

$L = 2.5\,m$ $\quad L = \frac{L_o}{\gamma}$ $\quad \gamma = \frac{L_o}{L} = \frac{7}{2.5} = 2.8$

$L_o = 7\,m$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \left(\sqrt{1-\beta^2}\right)^2 = \left(\frac{1}{\gamma}\right)^2 \Rightarrow 1-\beta^2 = \left(\frac{1}{\gamma}\right)^2$

$\beta^2 = 1 - \frac{1}{\gamma^2}$ $\quad \boxed{\beta = \sqrt{1-\frac{1}{\gamma^2}}} = 0.934\,c \leftarrow 3\times10^8\,m/s$

$v = 0.934 \times 3\times10^8\,m/s = 2.79\times10^8\,m/s$

7. Imagine you are a pole vaulter running at 0.80c with an 8.0m long pole towards a garage that is 8.0m deep.

a. How long will the pole look to a stationary observer?

b. How long will the garage look to you as you run in?

c. Will it fit?

a) $\gamma = 1.67$ $\quad L = ?$ $\quad L = \frac{L_o}{\gamma} = \frac{8\,m}{1.67} = 4.8\,m$

$L_o = 8\,m$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

b) $L = \frac{L_o}{\gamma} = \frac{8\,m}{1.67} = 4.8\,m$

c) Yes.

8. The closest star to the Earth is 4 light years away. 1 light year is the distance that light travels in 1 year.

a) At what speed would you have to travel so that you can get there in only 3 years according to the clock on board the space craft.

b) How long will the stationary observer say it took?

$d = vt$
$= c\,y$
$= Ly$

$T_o = 3\,y$ $\quad T = \gamma T_o = \gamma \cdot 3\,y$ $\quad \boxed{V = \frac{d}{T}}$

$d = 4\,Ly$ or $4\,c\cdot y$

$V = \frac{4\,cy}{3\gamma y} \Rightarrow V = \frac{4c}{3\gamma}$ and $\boxed{\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$

$d = vt$

$t = \frac{d}{v} = \gamma t_o$ $\quad \boxed{\left(\frac{d}{v}\right)^2 = \left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\,t_o\right)^2}$

$\frac{d^2}{v^2} = \frac{t_o^2}{\frac{c^2}{c^2} - \frac{v^2}{c^2}}$ $\quad \frac{d^2}{v^2} = \frac{t_o^2}{\frac{c^2-v^2}{c^2}}$ $\quad \frac{a}{\frac{c}{d}} \Rightarrow a \div \frac{c}{d}$ $\quad a \times \frac{d}{c}$

$\frac{d^2}{v^2} = \frac{t_o^2 c^2}{c^2 - v^2}$

$d^2c^2 - d^2v^2 = t_o^2 v^2 c^2$ $\quad d^2c^2 = \left(t_o^2 \cancel{v^2} c^2 + d^2 \cancel{v^2}\right) v^2$

$t_o = 3\,y$ $\quad t = \gamma t_o$
$t = 3\gamma\,y$ $\quad = 3\gamma\,y$
$L_o =$
$L = 4\,cy$

$d = v \cdot t$

$v = \frac{d}{t} = \frac{4\,cy}{3\gamma y}$

$\boxed{v = \frac{4c}{3\gamma}}$ or $\boxed{\gamma = \frac{4c}{3v}}$

$\beta = \frac{v}{c} = \frac{4\cancel{c}}{3\gamma\cancel{c}}$

$\beta = \frac{4}{3\gamma}$ $\quad$ Sub into our γ equations

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$$d^2c^2 - d^2v^2 = t_0^2 v^2 c^2 \Rightarrow d^2c^2 = (t_0^2 \cancel{v^2} c^2 + d^2 \cancel{v^2}) v^2$$

$$v^2(t_0^2c^2 + d^2) = d^2c^2$$

$$\sqrt{v^2} = \sqrt{\frac{d^2c^2}{t_0^2c^2+d^2}} \qquad v = \frac{dc}{\sqrt{t_0^2c^2+d^2}} = \frac{4c\!\cdot\!y\, c}{\sqrt{9c^2+16c^2y^2}} \qquad \frac{4c}{5}$$

$$= \frac{dc}{\sqrt{t_0^2c^2+d^2}} = \frac{4C}{\sqrt{9_y c^2 + 16c^2y^2}} = \frac{4C}{\sqrt{25c^2y^2}} = \frac{4C^2\cancel{y}}{5c\cancel{y}}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{\left(\frac{4c}{3\gamma}\right)^2}{c^2}}} \Rightarrow \frac{1}{\sqrt{\frac{1-16\cancel{c^2}}{\frac{9\gamma^2}{\cancel{c^2}}}}} \Rightarrow \frac{1}{\sqrt{1-\frac{16}{9\gamma^2}}}$$

$$(\gamma)^2 = \left(\frac{1}{\sqrt{1-\frac{16}{9\gamma^2}}}\right)^2 \Rightarrow \gamma^2 = \frac{1}{1-\frac{16}{9\gamma^2}}$$

$$\gamma^2 = \frac{1}{\frac{9\gamma^2-16}{9\gamma^2}} \Rightarrow \cancel{\gamma^2}\left(\frac{9\gamma^2-16}{9\cancel{\gamma^2}}\right) = 1$$

$$\Rightarrow 9\gamma^2 - 16 = 9 \qquad 9\gamma^2 = 25 \qquad \sqrt{\gamma^2} = \sqrt{\frac{25}{9}}$$

$\gamma = \frac{5}{3}$ therefore $V = \frac{4c}{3\gamma} = \frac{4c}{\cancel{3}\left(\frac{5}{\cancel{3}}\right)}$

$$\boxed{V = \frac{4}{5}c \text{ OR } 0.8c}$$

The time for an observer is $T = \gamma T_0 = \frac{5}{\cancel{3}}(\cancel{3}y) = 5$ years

This makes sense because $t = \frac{d}{v} = \frac{4\cancel{c}y}{4/5\cancel{c}} = 5$ years!

The distance travelled according to the spaceship is

$L = \frac{L_0}{\gamma} = \frac{4c\cdot y}{5/3} = \frac{12}{5}c\cdot y = 2.2\,c\cdot y$ (light years)

so it will take $t = \frac{d}{v} = \frac{12/\cancel{5}\,\cancel{c}y}{4/\cancel{5}\,\cancel{c}} = \frac{12}{4}y = 3$ years!
in the ship.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{4}{3\gamma}\right)^2}}$$

$$\left[\gamma^2 = \frac{1}{1-\frac{16}{9\gamma^2}}\right] \times \left(1 - \frac{16}{9\gamma^2}\right)$$

$$\gamma^2\left(1-\frac{16}{9\gamma^2}\right) = 1$$

$$\gamma^2 - \frac{16}{9} = 1$$

$$\sqrt{\gamma^2} = \sqrt{1+\frac{16}{9}}$$

$$\gamma = \frac{5}{3}$$

$$v = \frac{4c}{3\gamma} = \frac{4c}{3\left(\frac{5}{3}\right)}$$

$$\boxed{v = \frac{4}{5}c} \text{ or } 0.8c$$