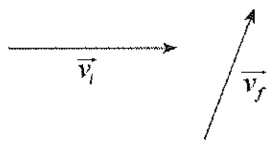
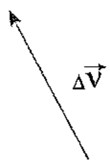
Physics 12 - Vector Practice
Answer Section

SHORT ANSWER

1. Initial velocity vector $\vec{v}_i$ and final velocity vector $\vec{v}_f$ are shown below.
   Which of the following represents the change in velocity $\Delta\vec{v}$? **(1 mark)**



(Answer: 1 mark)



b.

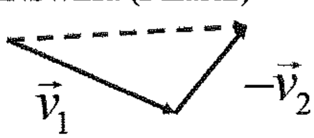
2. An aircraft is flying due south. Some time later it is flying due east. Which vector best represents the aircraft's change in velocity? **(1 mark)**
   (Answer:1 mark)

d.

3. Consider the two vectors shown below. **(2 marks)**

Please draw the resultant vector of $\vec{v}_1 - \vec{v}_2$

ANSWER: (2 marks)



4. Vector of $\vec{A}$ =25 m/s @ 39.5° S of W **and** Vector $\vec{B}$ = 38.5 m/s.@ 35° N of E.
   What is the resultant vector of $\vec{A}+\vec{B}$? **(3 marks)**

   ANSWER: (3 marks)

   $\vec{A}_X = 25\,m/s \times \cos(39.5°) = -19.29\,m/s$ $\vec{A}_Y = 25\,m/s \times \sin(39.5°) = -15.9\,m/s$

   $\vec{B}_X = 38.5\,m/s \times \cos(35°) = 31.54\,m/s$ $\vec{B}_Y = 38.5\,m/s \times \sin(35°) = 22.08\,m/s$

   $\vec{R}_X = \vec{A}_X + \vec{B}_X = -19.29\,m/s + 31.54\,m/s = 12.25\,m/s$

   $\vec{R}_Y = \vec{A}_Y + \vec{B}_Y = -15.9\,m/s + 22.08\,m/s = 6.18\,m/s$

   $$\vec{R} = \sqrt{\left(\vec{R}_X\right)^2 + \left(\vec{R}_Y\right)^2} = \sqrt{(12.25)^2 + (6.18)^2} = \underline{\underline{13.72 m/s}}$$

   The direction is $\tan^{-1}\left(\frac{\vec{R}_Y}{\vec{R}_X}\right) = \tan^{-1}\left(\frac{6.18}{12.25}\right) = \underline{\underline{26.8° N\,of\,E}}$

5. Vector of $\vec{A}$ =46 m @ 13° N of W **and** Vector $\vec{B}$ = 7 m.@ 59.5° W of S.
   What is the resultant vector of $\vec{A}-\vec{B}$? **(3 marks)**

   ANSWER: (3 marks)

   $\vec{A}_X = 46\,m \times \cos(13°) = -44.82\,m$ $\vec{A}_Y = 46\,m \times \sin(13°) = 10.35\,m$

   $-\vec{B}_X = 7\,m \times \sin(59.5°) = 6.03\,m$ $-\vec{B}_Y = 7\,m \times \cos(59.5°) = 3.55\,m$

   $\vec{R}_X = \vec{A}_X + \left(-\vec{B}_X\right) = -44.82\,m + (6.03\,m) = -38.79\,m$

   $\vec{R}_Y = \vec{A}_Y + \left(-\vec{B}_Y\right) = 10.35\,m + (3.55\,m) = 13.9\,m$

   $$\vec{R} = \sqrt{\left(\vec{R}_X\right)^2 + \left(\vec{R}_Y\right)^2} = \sqrt{(-38.79)^2 + (13.9)^2} = \underline{\underline{41.21m}}$$

   The direction is $\tan^{-1}\left(\frac{\vec{R}_Y}{\vec{R}_X}\right) = \tan^{-1}\left(\frac{13.9}{38.79}\right) = \underline{\underline{19.7° N\,of\,W}}$

6. Vector $\vec{A}$ =32.5 m/s @ 76° W of N **and** Vector $\vec{B}$ = 44.5 m/s.@ 31° W of N.
   a) What is the resultant vector of $\vec{A}+\vec{B}$ **(3 marks)**

ANSWER: (3 marks)
$\vec{A}+\vec{B}$

$\vec{A}_X = 32.5\,m/s \times \sin(76°) = -31.53\,m/s \qquad \vec{A}_Y = 32.5\,m/s \times \cos(76°) = 7.86\,m/s$

$\vec{B}_X = 44.5\,m/s \times \sin(31°) = -22.92\,m/s \qquad \vec{B}_Y = 44.5\,m/s \times \cos(31°) = 38.14\,m/s$

$\vec{R}_X = \vec{A}_X + \vec{B}_X = -31.53\,m/s + -22.92\,m/s = -54.45\,m/s$

$\vec{R}_Y = \vec{A}_Y + \vec{B}_Y = 7.86\,m/s + 38.14\,m/s = 46.01\,m/s$

$$\vec{R} = \sqrt{\left(\vec{R}_X\right)^2 + \left(\vec{R}_Y\right)^2} = \sqrt{(-54.45)^2 + (46.01)^2} = \underline{\underline{71.29 m/s}}$$

The direction is $\tan^{-1}\left(\frac{\vec{R}_Y}{\vec{R}_X}\right) = \tan^{-1}\left(\frac{46.01}{54.45}\right) = \underline{\underline{40.2° N\ of\ W}}$

b) What is the resultant vector of $\vec{A}-\vec{B}$ **(2 marks)**

ANSWER: (2 marks)
$\vec{A}-\vec{B}$

$\vec{A}_X = 32.5\,m/s \times \sin(76°) = -31.53\,m/s \qquad \vec{A}_Y = 32.5\,m/s \times \cos(76°) = 7.86\,m/s$

$-\vec{B}_X = 44.5\,m/s \times \sin(31°) = 22.92\,m/s \qquad -\vec{B}_Y = 44.5\,m/s \times \cos(31°) = -38.14\,m/s$

$\vec{R}_X = \vec{A}_X + \left(-\vec{B}_X\right) = -31.53\,m/s + (22.92\,m/s) = -8.62\,m/s$

$\vec{R}_Y = \vec{A}_Y + \left(-\vec{B}_Y\right) = 7.86\,m/s + (-38.14\,m/s) = -30.28\,m/s$

$$\vec{R} = \sqrt{\left(\vec{R}_X\right)^2 + \left(\vec{R}_Y\right)^2} = \sqrt{(-8.62)^2 + (-30.28)^2} = \underline{\underline{31.48 m/s}}$$

The direction is $\tan^{-1}\left(\frac{\vec{R}_Y}{\vec{R}_X}\right) = \tan^{-1}\left(\frac{30.28}{8.62}\right) = \underline{\underline{74.1°\ S\ of\ W}}$

7. A pilot leaves their city at 12:30 pm and WANTS to fly West to an airport 1515 km away. They need to arrive at the airport at 6:00 pm.
ANSWER: (3 marks)
a) What is their airspeed and direction if there is a 39km/h wind blowing 44.5°N of E. **(3 marks)**

First we need to determine the ground speed of the aircraft.

Divide the distance of the airports by the time needed to be in the air.

$$V_{groundspeed} = \frac{d_{airports}}{t_{air}} = \frac{1515km}{5.5h} = 275.45km/h$$

Now we need to determine the velocity components.

$V_{Ground_X} = 275.45$ km/h $West = -275.45$ km/h $\quad V_{Ground_Y} = 0$ km/h

$V_{Wind_X} = 39$ km/h $\times \cos(44.5°) = 27.82$ km/h $\quad V_{Wind_Y} = 39$ km/h $\times \sin(44.5°) = 27.34$ km/h

The groundspeed is the resultant vector of adding the airspeed to the windspeed,

$V_{Ground} = V_{Air} + V_{Wind}$ So $V_{Air} = V_{Ground} - V_{Wind}$

$V_{Air_X} = V_{Ground_X} - V_{Wind_X} = -275.45$ km/h $-$ (27.82 km/h) $= -303.27$ km/h

$V_{Air_Y} = V_{Ground_Y} - V_{Wind_Y} = 0$ km/h $-$ (27.34 km/h) $= -27.34$ km/h

$$\vec{R} = \sqrt{\left(\vec{R}_X\right)^2 + \left(\vec{R}_Y\right)^2} = \sqrt{(-303.27)^2 + (-27.34)^2} = \underline{\underline{304.5km/h}}$$

The direction is $\tan^{-1}\left(\frac{\vec{R}_Y}{\vec{R}_X}\right) = \tan^{-1}\left(\frac{27.34}{303.27}\right) = \underline{\underline{5.2°S\ of\ W}}$

b) What is the minimum amount of fuel they need to carry if their plane burns 128 litres/h of fuel for every 100 km/h they fly?**(1 mark)**
To find the amount of fuel needed, multiply the the fuel rate by the airspeed of the plane and divide by 100.

$$\text{amount of fuel} = \frac{128l/h \times 304.5km/h}{100km/h} = 128l/h \times 5.5h = \underline{\underline{2143.7l}}$$

8. A swimmer wants to swim North across a 228 metres wide river.
a) How much time does it take them to cross a river if they can swim at 9.5 m/s in still water but the river has a current of 6 m/s E? **(1 mark)**
ANSWER: (1 mark)
a) $t = \frac{d}{v} = \frac{228\,m}{9.5\,m/s} = \underline{\underline{24s}}$

b) How far downstream from the point directly opposite the starting point will the swimmer get out of the water after having crossed the river? **(1 mark)**
ANSWER: (1 mark)
b) $d = v \times t = 6\,m/s \times 24\,s = \underline{\underline{144m}}$

c) The swimmer wants to reach the other side directly opposite the start point
What is their relative speed to someone standing on shore?
And what direction do they need to head?
ANSWER: (1 mark)
c) $relative\ velocity = \sqrt{velocity^2 - current^2} = \sqrt{(9.5)^2 - (6)^2} = \underline{\underline{7.37m/s}}$

$$Angle = \sin^{-1}\left(\frac{current}{velocity}\right) = \sin^{-1}\left(\frac{6}{9.5}\right) = \underline{\underline{39.2^\circ\ W\,of\,N}}$$

d) How long would it take them to cross? **(1 mark)**
ANSWER: (1 mark)
d) $t = \frac{d}{v} = \frac{228}{7.37} = \underline{\underline{30.96s}}$