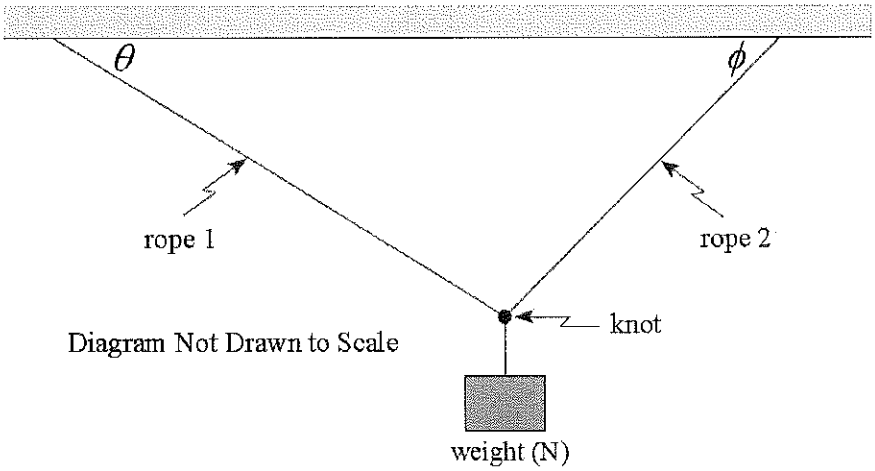
Name: _____ Class: _____ Date: _____ ID: A

Equilibrium part 1

Short Answer

1. A 228N weight is supported by two ropes fastened together by a knot, as shown in the diagram below. The angle $\theta = 38°$ and the angle $\phi = 22°$ **(4 marks)**



a) What is the tension in rope 1? **(2 marks)**

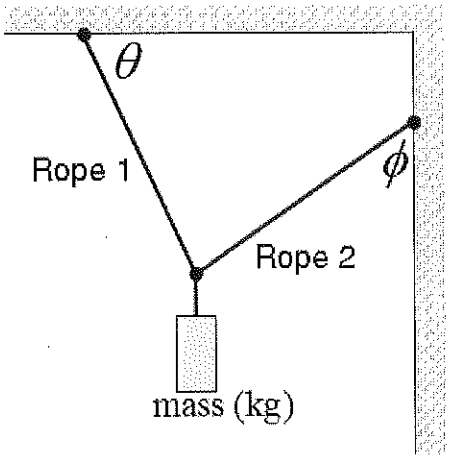
ANSWER: _____

b) What is the tension in rope 2? **(2 marks)**

ANSWER: _____

Name: _____ ID: A

2. A 75kg mass is supported by two ropes fastened together by a knot, as shown in the diagram below. The angle $\theta = 59°$ and the angle $\phi = 69°$ shown suspended by two ropes. **(4 marks)**



a) What is the tension in rope 1? **(2 marks)**

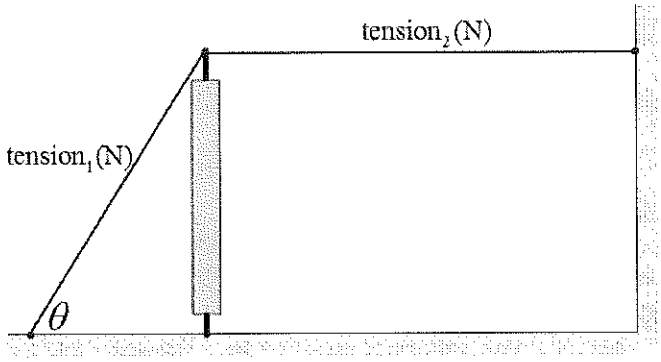
ANSWER: _____

b) What is the tension in rope 2? **(2 marks)**

ANSWER: _____

Name: _____ ID: A

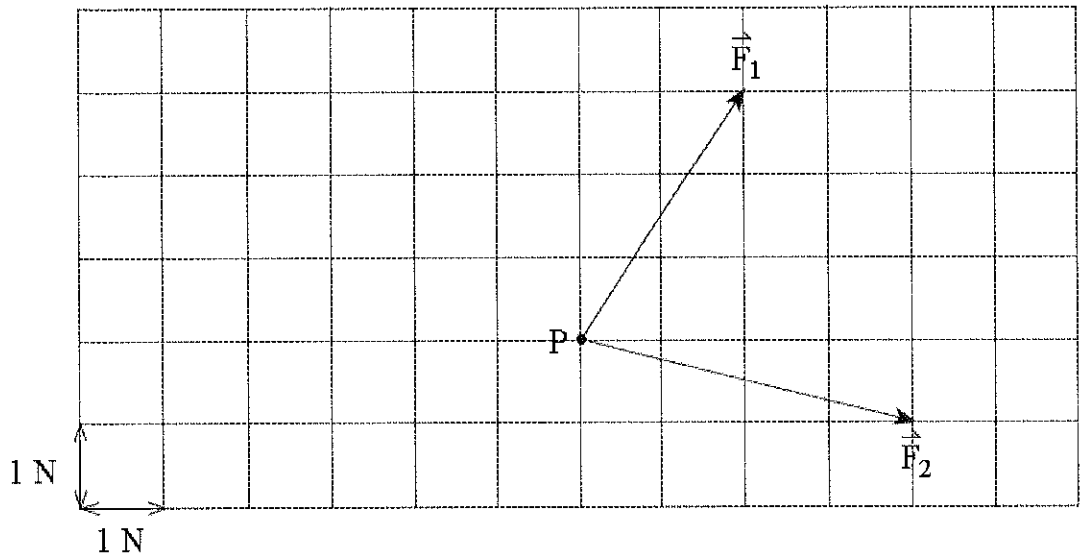
3. Two cables support a vertical tower. The tension in each cable is shown.
$tension_1 = 323N$ $tension_2 = 64N$



What is the angle θ? **(3 marks)**

ANSWER: _____

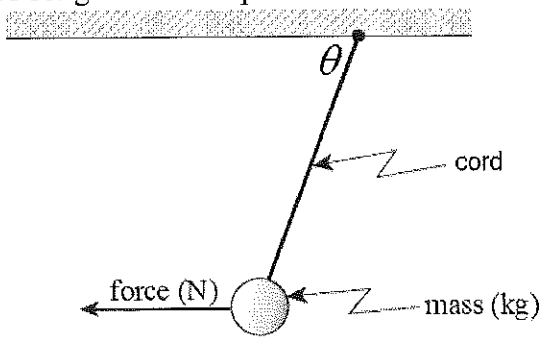
4. Two forces act at point P as shown below.



Find the magnitude of the third force required to achieve equilibrium.

ANSWER: _____

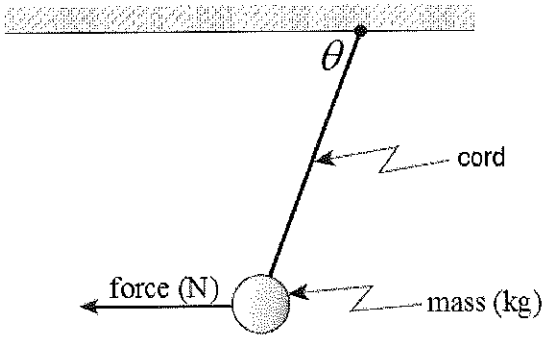
5. A 11kg mass is suspended from a cord at an angle of 23°.



What force is needed to hold it at that angle? **(3 marks)**

ANSWER: _____

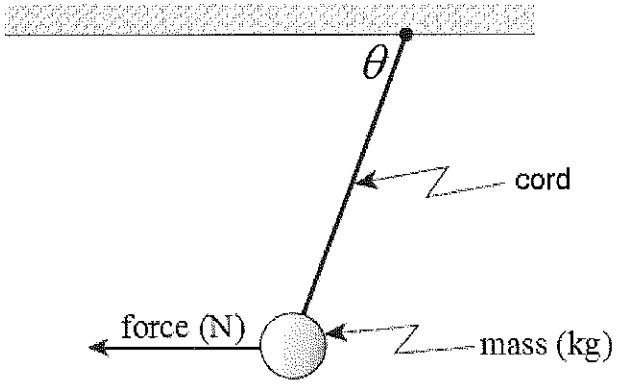
6. A mass is suspended from a cord at an angle of 64°. The mass is held as shown with a horizontal 498N force.



Find the mass. **(3 marks)**

ANSWER: _____

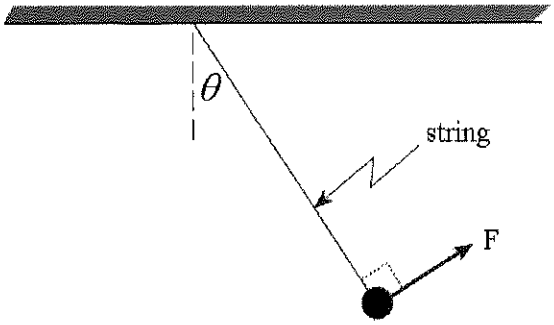
7. A 21kg mass is suspended from a cord. The mass is held as shown with a horizontal 163N force.



Find the angle θ. **(3 marks)**

ANSWER: _____

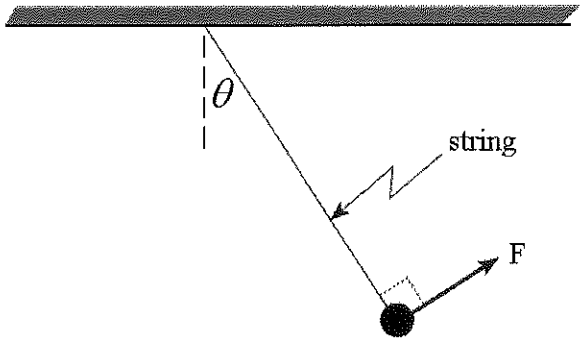
8. A 43kg mass suspended by a string is held 43° from vertical as shown.



Find the force.

ANSWER: _____

9. A mass suspended by a string is held 39° from vertical by a force of 408 N as shown.

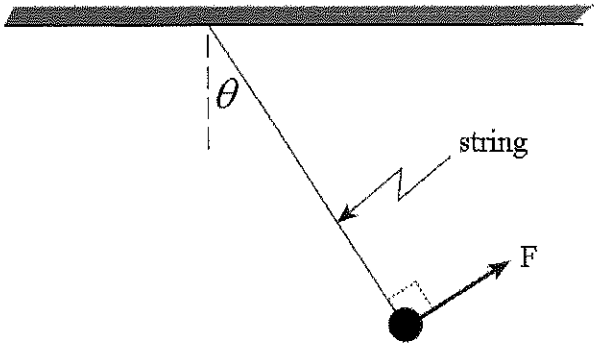


Find the mass.

ANSWER:

_____

10. A 11kg mass suspended by a string is held at an angle θ from vertical as shown by a force of 52N.



Find the angle.

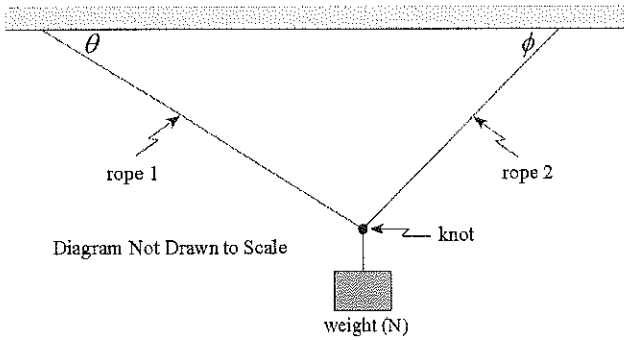
ANSWER:

_____

Equilibrium part 1
Answer Section

SHORT ANSWER

1. A 228N weight is supported by two ropes fastened together by a knot, as shown in the diagram below. The angle $\theta = 38°$ and the angle $\phi = 22°$ **(4 marks)**



ANSWER: (4 marks)

a) What is the tension in rope 1? **(2 marks)**

$\sum F_x = 0$

$-T_{x_1} + T_{x_2} = 0$

$-T_1 \cdot \cos(\theta) + T_2 \cdot \cos(\phi) = 0$

$T_2 = \dfrac{T_1 \cdot \cos(\theta)}{\cos(\phi)}$

$\sum F_y = 0$

$T_{y_1} + T_{y_2} - W = 0$

$T_1 \cdot \sin(\theta) + T_2 \cdot \sin(\phi) = W$

$T_1 \cdot \sin(\theta) + \dfrac{T_1 \cdot \cos(\theta)}{\cos(\phi)} \cdot \sin(\phi) = W$

$T_1 \left(\sin(\theta) + \dfrac{\cos(\theta)}{\cos(\phi)} \cdot \sin(\phi) \right) = W$

$T_1 = \dfrac{W}{\left(\sin(\theta) + \dfrac{\cos(\theta)}{\cos(\phi)} \cdot \sin(\phi) \right)} = \dfrac{228N}{\left(\sin(38) + \dfrac{\cos(38)}{\cos(22)} \cdot \sin(22) \right)} = \underline{\underline{244.1N}}$

b) What is the tension in rope 2? **(2 marks)**

$$\sum F_x = 0$$

$$-T_{x_1} + T_{x_2} = 0$$

$$-T_1 \cdot \cos(\theta) + T_2 \cdot \cos(\phi) = 0$$

$$T_1 = \frac{T_2 \cdot \cos(\phi)}{\cos(\theta)}$$

$$\sum F_y = 0$$

$$T_{y_1} + T_{y_2} - W = 0$$

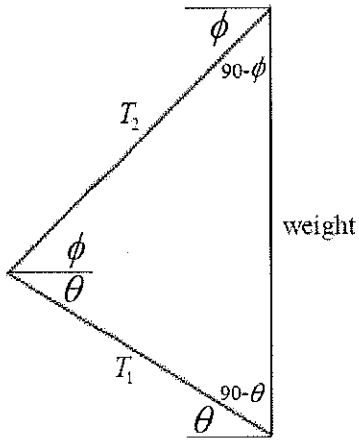
$$T_1 \cdot \sin(\theta) + T_2 \cdot \sin(\phi) = W$$

$$\frac{T_2 \cdot \cos(\phi)}{\cos(\theta)} \cdot \sin(\theta) + T_2 \cdot \sin(\phi) = W$$

$$T_2 \left(\frac{\cos(\phi)}{\cos(\theta)} \cdot \sin(\theta) + \sin(\phi) \right) = W$$

$$T_2 = \frac{W}{\left(\frac{\cos(\phi)}{\cos(\theta)} \cdot \sin(\theta) + \sin(\phi) \right)} = \frac{228N}{\left(\frac{\cos(22°)}{\cos(38°)} \cdot \sin(38°) + \sin(22°) \right)} = \underline{\underline{207.46N}}$$

OR using the geometry method:



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

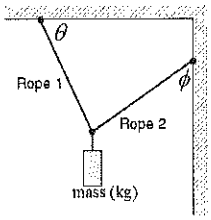
$$\frac{T_1}{\sin(90° - \phi)} = \frac{weight}{\sin(\theta + \phi)}$$

$$T_1 = \frac{weight \cdot \sin(90° - \phi)}{\sin(\theta + \phi)} = \frac{228N \cdot \sin(90° - 22°)}{\sin(38° + 22°)} = \underline{\underline{244.1N}}$$

$$\frac{T_2}{\sin(90° - \theta)} = \frac{weight}{\sin(\theta + \phi)}$$

$$T_2 = \frac{weight \cdot \sin(90° - \theta)}{\sin(\theta + \phi)} = \frac{228N \cdot \sin(90° - 38°)}{\sin(38° + 22°)} = \underline{\underline{207.46N}}$$

2. A 75kg mass is supported by two ropes fastened together by a knot, as shown in the diagram below. The angle $\theta = 59°$ and the angle $\phi = 69°$ shown suspended by two ropes. **(4 marks)**



ANSWER: (4 marks)

a) What is the tension in rope 1? **(2 marks)**

$\sum F_x = 0$

$-T_{x_1} + T_{x_2} = 0$

$-T_1 \cdot \cos(\theta) + T_2 \cdot \sin(\phi) = 0$

$T_2 = \dfrac{T_1 \cdot \cos(\theta)}{\sin(\phi)}$

$\sum F_y = 0$

$T_{y_1} + T_{y_2} - W = 0$

$T_1 \cdot \sin(\theta) + T_2 \cdot \cos(\phi) = m \cdot 9.8m/s^2$

$T_1 \cdot \sin(\theta) + \dfrac{T_1 \cdot \cos(\theta)}{\sin(\phi)} \cdot \cos(\phi) = m \cdot 9.8m/s^2$

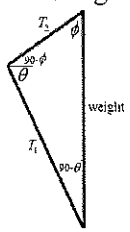
$T_1 \left(\sin(\theta) + \dfrac{\cos(\theta)}{\sin(\phi)} \cdot \cos(\phi) \right) = m \cdot 9.8m/s^2$

$$T_1 = \frac{m \cdot 9.8m/s^2}{\left(\sin(\theta) + \dfrac{\cos(\theta)}{\sin(\phi)} \cdot \cos(\phi) \right)} = \frac{75kg \cdot 9.8m/s^2}{\left(\sin(59) + \dfrac{\cos(59)}{\sin(69)} \cdot \cos(69) \right)} = \underline{\underline{696.77N}}$$

b) What is the tension in rope 2? **(2 marks)**

$$T_2 = \frac{T_1 \cdot \cos(\theta)}{\sin(\phi)} = \frac{696.77N \cdot \cos(59°)}{\sin(69°)} = \underline{\underline{384.39N}}$$

OR using the geometry method:



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

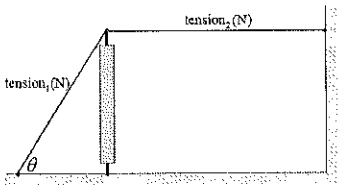
$$\frac{T_1}{\sin(\phi)} = \frac{weight}{\sin(\theta + 90° - \phi)}$$

$$T_1 = \frac{m \cdot 9.8m/s^2 \cdot \sin(\phi)}{\sin(\theta + 90° - \phi)} = \frac{75kg \cdot 9.8m/s^2 \cdot \sin(69°)}{\sin(59° + 90° - 69°)} = \underline{\underline{696.77N}}$$

$$\frac{T_2}{\sin(90° - \theta)} = \frac{weight}{\sin(\theta + 90° - \phi)}$$

$$T_2 = \frac{m \cdot 9.8m/s^2 \cdot \sin(90° - \theta)}{\sin(\theta + 90° - \phi)} = \frac{75kg \cdot 9.8m/s^2 \cdot \sin(90° - 59°)}{\sin(59° + 90° - 69°)} = \underline{\underline{384.39N}}$$

3. Two cables support a vertical tower. The tension in each cable is shown.
$tension_1 = 323N$ $tension_2 = 64N$



What is the angle θ? **(3 marks)**

ANSWER: (3 marks)

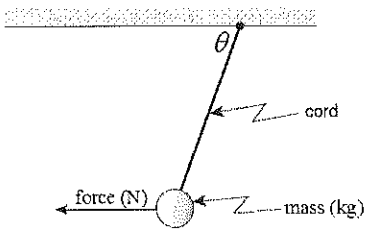
$\sum F_x = 0$

$-T_{x_1} + T_2 = 0$ $\quad T_{x_1} = T_2$

$$\cos(\theta) = \frac{T_{x_1}}{T_1} \quad T_{x_1} = T_2$$

$$\cos(\theta)^{-1} = \frac{T_2}{T_1} = \frac{64N}{323N} = \underline{\underline{78.57°}}$$

4. adsfds



5. A 11kg mass is suspended from a cord at an angle of 23°. What force is needed to hold it at that angle? **(3 marks)**

ANSWER: (3 marks)

$\sum F_x = 0$

$-F_{x_L} + F_{x_R} = 0 \qquad F_{x_L} = F_{x_R}$

$\sum F_y = 0$

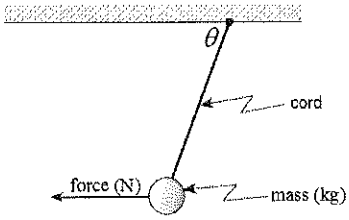
$F_y - F_g = 0 \qquad F_y = F_g \qquad \tan(\theta) = \dfrac{F_y}{F_{x_R}} \qquad F_{x_L} = F_{x_R}$

$F_y = F_{x_L} \cdot \tan(\theta)$

$F_{x_L} \cdot \tan(\theta) = m \cdot g$

$F_{x_L} = \dfrac{m \cdot g}{\tan(\theta)} = \dfrac{11kg \cdot 9.8m/s^2}{\tan(23°)} = \underline{\underline{253.96N}}$

6. A mass is suspended from a cord at an angle of 64°. The mass is held as shown with a horizontal 498N force.



Find the mass. **(3 marks)**

ANSWER: (3 marks)

$\sum F_x = 0$

$-F_{x_L} + F_{x_R} = 0 \qquad F_{x_L} = F_{x_R}$

$\sum F_y = 0$

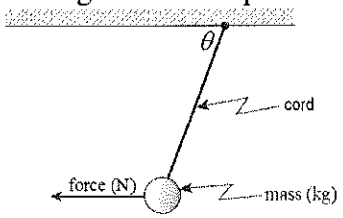
$F_y - F_g = 0 \qquad F_y = F_g \qquad \tan(\theta) = \dfrac{F_y}{F_{x_R}} \qquad F_{x_L} = F_{x_R}$

$F_y = F_{x_L} \cdot \tan(\theta)$

$F_{x_L} \cdot \tan(\theta) = m \cdot g$

$m = \dfrac{F_{x_L} \cdot \tan(\theta)}{g} = \dfrac{498N \cdot \tan(64°)}{9.8m/s^2} = \underline{\underline{104.19kg}}$

7. A 21kg mass is suspended from a cord. The mass is held as shown with a horizontal 163N force.



Find the angle θ. **(3 marks)**

ANSWER: (3 marks)

$\sum F_x = 0$

$-F_{x_L} + F_{x_R} = 0 \qquad F_{x_L} = F_{x_R}$

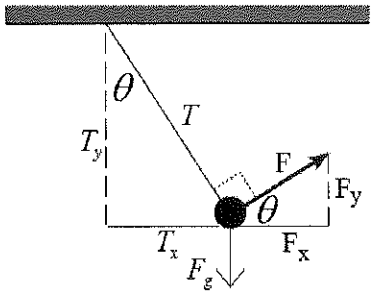
$\sum F_y = 0$

$F_y - F_g = 0 \qquad F_y = F_g \qquad \tan(\theta) = \dfrac{F_y}{F_{x_R}} \qquad F_{x_L} = F_{x_R}$

$F_y = F_{x_L} \cdot \tan(\theta)$

$F_{x_L} \cdot \tan(\theta) = m \cdot g$

$$\tan(\theta)^{-1} = \frac{m \cdot g}{F_{x_L}} = \frac{21kg \cdot 9.8m/s^2}{163N} = \underline{\underline{51.62^\circ}}$$



8.

$\sum F_y = 0$

$T_y + F_y - F_g = 0$

$T\cos\theta + F\sin\theta - mg = 0$

Isolate T

$$T = \frac{mg - F\sin\theta}{\cos\theta}$$

$\sum F_x = 0$

$-T_x + F_x = 0$

$-T\sin\theta + F\cos\theta = 0$

Isolate T

$$T = \frac{F\cos\theta}{\sin\theta}$$

$$\frac{mg - F\sin\theta}{\cos\theta} = \frac{F\cos\theta}{\sin\theta}$$

Solve for F

$$\frac{mg}{\cos\theta} - \frac{F\sin\theta}{\cos\theta} = \frac{F\cos\theta}{\sin\theta}$$

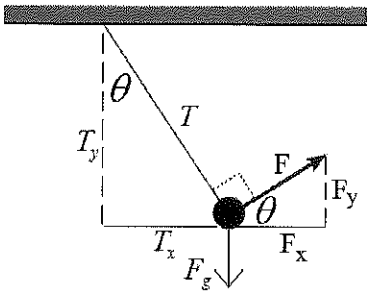
$$\frac{mg}{\cos\theta} = \frac{F\cos\theta}{\sin\theta} + \frac{F\sin\theta}{\cos\theta}$$ factor out F

$$F\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right) = \frac{mg}{\cos\theta}$$

$$F\left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cdot \cos\theta}\right) = \frac{mg}{\cos\theta}$$ Trigonometry Identity: $\cos^2\theta + \sin^2\theta = 1$

$$F\left(\frac{1}{\sin\theta \cdot \cos\theta}\right) = \frac{mg}{\cos\theta}$$ Cancel the cosine θ

$F = \sin\theta \cdot mg = \sin 43° \cdot 43kg \cdot 9.8m/s^2 = \underline{287.39N}$



9.

$\sum F_y = 0$

$T_y + F_y - F_g = 0$

$T\cos\theta + F\sin\theta - mg = 0$

Isolate T

$$T = \frac{mg - F\sin\theta}{\cos\theta}$$

$\sum F_x = 0$

$-T_x + F_x = 0$

$-T\sin\theta + F\cos\theta = 0$

Isolate T

$$T = \frac{F\cos\theta}{\sin\theta}$$

$$\frac{mg - F\sin\theta}{\cos\theta} = \frac{F\cos\theta}{\sin\theta}$$

Solve for m

$$\frac{mg}{\cos\theta} - \frac{F\sin\theta}{\cos\theta} = \frac{F\cos\theta}{\sin\theta}$$

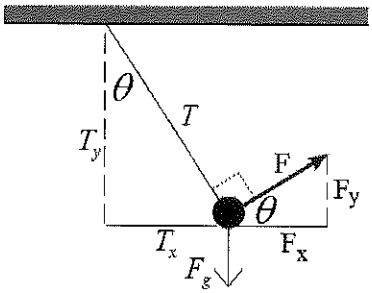
$$\frac{mg}{\cos\theta} = \frac{F\cos\theta}{\sin\theta} + \frac{F\sin\theta}{\cos\theta}$$ factor out F

$$F\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right) = \frac{mg}{\cos\theta}$$

$$F\left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cdot \cos\theta}\right) = \frac{mg}{\cos\theta}$$ Trigonometry Identity: $\cos^2\theta + \sin^2\theta = 1$

$$F\left(\frac{1}{\sin\theta \cdot \cos\theta}\right) = \frac{mg}{\cos\theta}$$ Cancel the cosine θ

$$m = \frac{F}{g\sin\theta} = \frac{408N}{\sin 39° \cdot 9.8m/s^2} = \underline{\underline{66.15kg}}$$



10.

$\sum F_y = 0$

$T_y + F_y - F_g = 0$

$T\cos\theta + F\sin\theta - mg = 0$

Isolate T

$$T = \frac{mg - F\sin\theta}{\cos\theta}$$

$\sum F_x = 0$

$-T_x + F_x = 0$

$-T\sin\theta + F\cos\theta = 0$

Isolate T

$$T = \frac{F\cos\theta}{\sin\theta}$$

$$\frac{mg - F\sin\theta}{\cos\theta} = \frac{F\cos\theta}{\sin\theta}$$

Solve for θ

$$\frac{mg}{\cos\theta} - \frac{F\sin\theta}{\cos\theta} = \frac{F\cos\theta}{\sin\theta}$$

$$\frac{mg}{\cos\theta} = \frac{F\cos\theta}{\sin\theta} + \frac{F\sin\theta}{\cos\theta}$$ factor out F

$$F\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right) = \frac{mg}{\cos\theta}$$

$$F\left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cdot\cos\theta}\right) = \frac{mg}{\cos\theta}$$ Trigonometry Identity: $\cos^2\theta + \sin^2\theta = 1$

$$F\left(\frac{1}{\sin\theta\cdot\cos\theta}\right) = \frac{mg}{\cos\theta}$$ Cancel the cosine θ

$$\sin\theta = \frac{F}{mg} = \frac{52N}{11kg\cdot 9.8m/s^2} \qquad \sin^{-1}\left(\frac{52N}{11kg\cdot 9.8m/s^2}\right) = \underline{\underline{28.84°}}$$