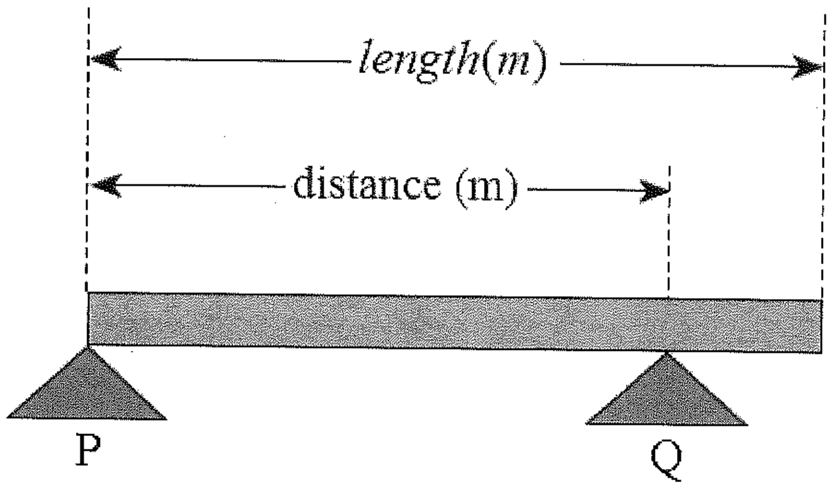
Equilibrium Part 2
Answer Section

SHORT ANSWER

1. A uniform beam of mass 12 kg, length 10 m, rests on supports P and Q, as shown in the diagram below. The distance between the supports is 9.5 m.



ANSWER: (4 marks)

a) What force is exerted by support Q on the beam? **(2 marks)**

$a)\tau_{cw} = \tau_{ccw}$

$rF_g = rF_Q$

$\frac{l}{2}mg = d \cdot F_Q$

$F_Q = \frac{mg}{2d} = \frac{12kg \cdot 9.8m/s^2}{2 \cdot 9.5m} = \underline{\underline{61.89N}}$

b) What force is exerted by support P on the beam? **(2 marks)**

$b)\tau_{cw} = \tau_{ccw}$

$r \cdot F_P = rF_g$

$d \cdot F_P = (d - \frac{l}{2}) \cdot mg$

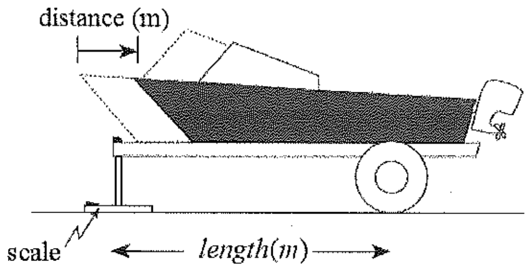
$F_P = \frac{\left(d - \frac{l}{2}\right) \cdot mg}{d} = \frac{\left(9.5m - \frac{10m}{2}\right) \cdot 12kg \cdot 9.8m/s^2}{9.5m} = \underline{\underline{55.71N}}$

$F_P + F_Q = mg$

$55.71N + 61.89N = 12kg \cdot 9.8m/s^2$

$\underline{\underline{117.6N = 117.6N}}$

2. A trailer carrying a boat is supported by a scale which initially reads 84 kg. The boat (and therefore its centre of gravity) is moved 0.3 m further back on the trailer. The scale now reads 28 kg. The length of the trailer is 6.1m.



Find the mass of the boat. **(3 marks)**
ANSWER: (3 marks)

$\tau_{cw} = \tau_{ccw}$

'r' is the distace to the center of mass

First Equation: $l \cdot F_{scale_1} = r \cdot m \cdot g$

Isolate 'r' $r = \dfrac{l \cdot F_{scale_1}}{m \cdot g}$

Second Equation: $l \cdot F_{scale_2} = (r - d) \cdot m \cdot g$

Substitute 'r' into the second equation. $l \cdot F_{scale_2} = \left(\dfrac{l \cdot F_{scale_1}}{m \cdot g} - d \right) \cdot m \cdot g$

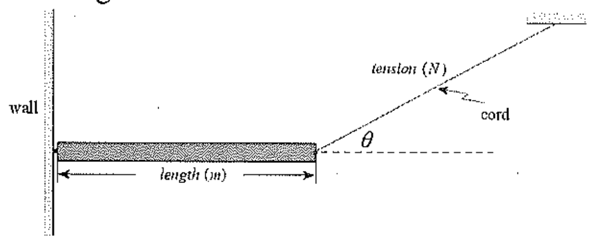
Solve for m. $l \cdot F_{scale_2} = \dfrac{l \cdot F_{scale_1} \cdot m \cdot g}{m \cdot g} - d \cdot m \cdot g$

$$\left(l \cdot F_{scale_2}\right) - \left(l \cdot F_{scale_1}\right) = -d \cdot m \cdot g$$

$$m = \frac{\left(l \cdot F_{scale_1}\right) - \left(l \cdot F_{scale_2}\right)}{d \cdot g} = \frac{l\left(F_{scale_1} - F_{scale_2}\right)}{d \cdot g} = \frac{6.1m\left(84kg - 28kg\right)}{0.3m \cdot 9.8m/s^2} = 116.19 \text{~~kg~~} \times 9.8$$

$= 1139\ kg$

3. A uniform 35kg bar, length 5.3m, is suspended by a cord as shown at an angle above the horizontal creating a tension of 250N.



What is the angle of the cord? **(3 marks)**
ANSWER: (3 marks)

$$\tau_{cw} = \tau_{ccw}$$

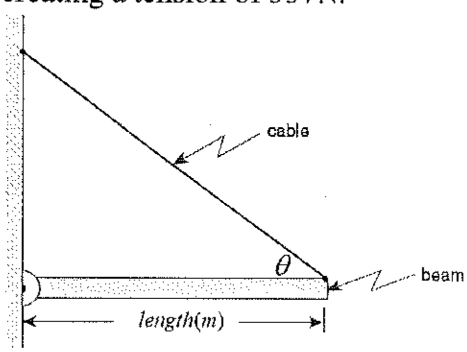
$$rF_g = rT_{\perp}$$

$$r \cdot mg = r \cdot T \cdot \sin(\theta)$$

$$2.65m \cdot 35kg \cdot 9.8m/s^2 = 5.3m \cdot T \cdot \sin(\theta)$$

$$\theta = \sin^{-1}\left(\frac{2.65m \cdot 35kg \cdot 9.8m/s^2}{5.3m \cdot 250N}\right) = \underline{\underline{43.31^\circ}}$$

4. A uniform bar, 5.4 m long, is suspended by a cord as shown at an angle of 67° above the horizontal creating a tension of 337N.



What is the mass of the uniform bar? **(3 marks)**
ANSWER: (3 marks)

$$\tau_{cw} = \tau_{ccw}$$

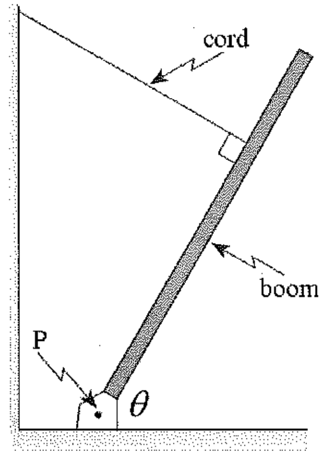
$$rF_g = rT_{\perp}$$

$$\frac{l}{2} \cdot mg = l \cdot T \cdot \sin(\theta)$$

$$2.7m \cdot (m) \cdot 9.8m/s^2 = 5.4m \cdot 337N \cdot \sin(67^\circ)$$

$$m = \frac{5.4m \cdot 337N \cdot \sin(67^\circ)}{2.7m \cdot 9.8m/s^2} = \underline{\underline{63.31kg}}$$

5. A 17m boom hinged at P is held stationary, as shown in the diagram below. The boom weighs 2,668N and makes an angle of 70° to the ground.



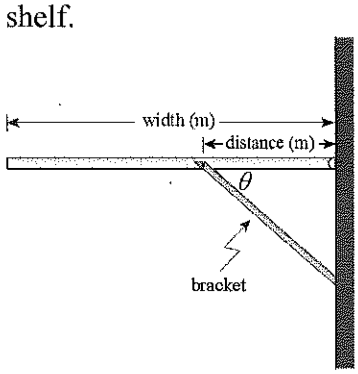
The supporting cord has a tension of 488N.
How far up the boom is the cord attached? **(3 marks)**
ANSWER: (3 marks)

$$\tau_{cw} = \tau_{ccw}$$

$$\frac{l}{2} W\cos(\theta) = d \cdot T$$

$$d = \frac{l \cdot W \cdot \cos(\theta)}{2 \cdot T} = \frac{17m \cdot 2{,}668N \cdot \cos(70^\circ)}{2 \cdot 488N} = \underline{\underline{15.89m}}$$

6. A shelf of width 1.8m is supported by a bracket attached at a distance of 0.4m, as shown in the diagram below. The bracket makes an angle of 33°with the shelf and exerts a force of 8N on the shelf.



What is the mass of the shelf? **(3 marks)**
ANSWER: (3 marks)

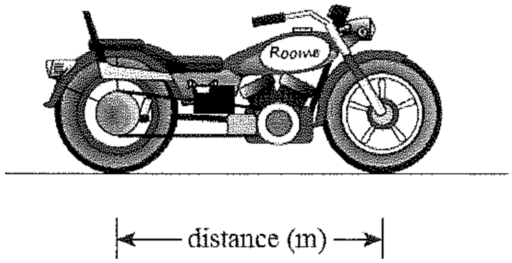
$$\tau_{cw} = \tau_{ccw}$$

$$d \cdot F \cdot \sin(\theta) = \frac{w}{2} \cdot m \cdot g$$

$$m = \frac{2 \cdot d \cdot F \cdot \sin(\theta)}{w \cdot g} = \frac{2 \cdot 0.4m \cdot 8N \cdot \sin(33^\circ)}{1.8m \cdot 9.8m/s^2} = ~~18.98kg~~$$

0.198kg

7. The motorcycle shown has a mass of 424 kg and a wheel base of 1.4 m.



If the rear wheel exerts a 392 N force on the ground, find how far the motorcycle's centre of gravity is located from the front wheel. **(3 marks)**

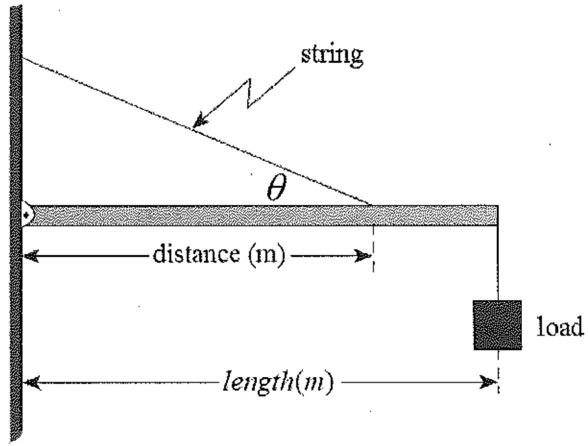
ANSWER: (3 marks)

$$\tau_{cw} = \tau_{ccw}$$

$$d \cdot F_{rear} = r \cdot m \cdot g$$

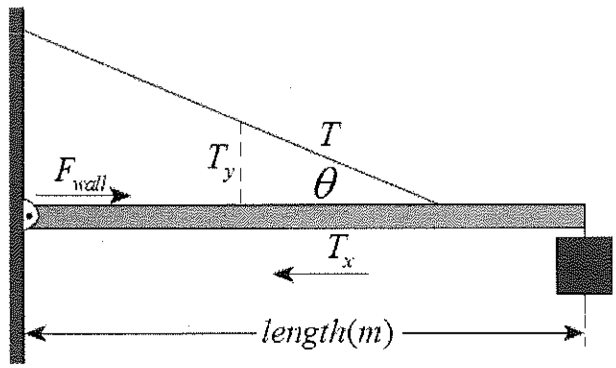
$$r = \frac{d \cdot F_{rear}}{m \cdot g} = \frac{1.4m \cdot 392N}{424kg \cdot 9.8m/s^2} = \underline{\underline{0.13m}}$$

8. The diagram shows a 14.4m horizontal beam. The wall exerts a 157N horizontal force on the beam. The string makes an angle of 67° with the beam. There is a load of mass 38kg hanging from the end.



distance = 12m

Find the mass of the beam. **(3 marks)**
ANSWER: (3 marks)



$F_{wall} = T_x$ and $\tan(\theta) = \dfrac{T_y}{T_x}$ so $T_\perp = T_y = F_{wall} \cdot \tan(\theta)$

$\tau_{cw} = \tau_{ccw}$

$\dfrac{l}{2} F_{beam} + l \cdot F_{load} = dT_\perp$

$\dfrac{l}{2} \cdot m_{beam} \cdot g + l \cdot m_{load} \cdot g = d \cdot F_{wall} \cdot \tan(\theta)$ $\quad m_{beam} = \dfrac{2\left(d \cdot F_{wall} \cdot \tan(\theta) - l \cdot m_{load} \cdot g\right)}{l \cdot g}$

$$m_{beam} = \frac{2\left(12m \cdot 157N \cdot \tan(67°) - \frac{14.4m}{} \cdot 38kg \cdot 9.8m/s^2\right)}{14.4m \cdot 9.8m/s^2} = ~~24.9kg~~$$

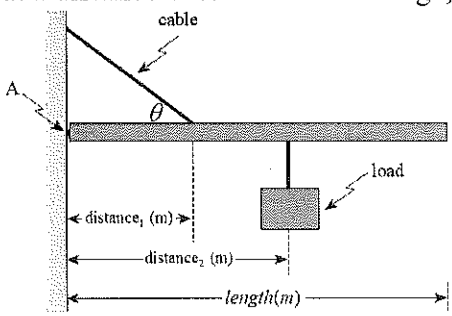
(249 crossed out and replaced by 38)

−13.097...

−13 kg

This question does not work with the variables given. 38kg should be 24.4kg giving m_{beam} = 13kg

9. A uniform beam 13.4m long, and with a mass of 22kg, is hinged at A. The supporting cable, located at a distance 10.8m from the hinge, is at an angle of 54°.



If the maximum tension the cable can withstand is 1,145N, what is the maximum mass of the load if it is located at 7.5m from the hinge? **(3 marks)**

ANSWER: (3 marks)

$$\tau_{cw} = \tau_{ccw}$$

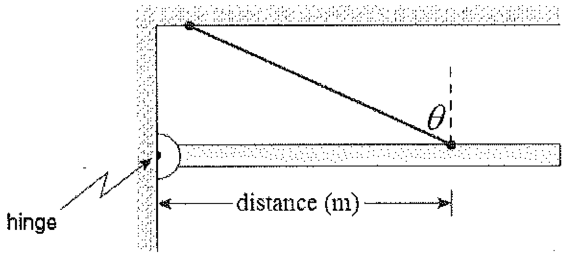
$$d_{beam}F_{beam} + d_{load}F_{load} = d_{cable}T_{\perp}$$

$$\frac{l}{2}m_{beam}g + d_{load}m_{load}g = d_{cable}T\sin(\theta)$$

$$m_{load} = \frac{d_{cable} \cdot T \cdot \sin(\theta) - \frac{l}{2} \cdot m_{beam} \cdot g}{d_{load} \cdot g} = \frac{10.8m \cdot 1,145N \cdot \sin(54°) - \frac{13.4m}{2} \cdot 22 \cdot 9.8m/s^2}{7.5m \cdot 9.8m/s^2}$$

$$m_{load} = \underline{\underline{116.46kg}}$$

10. A uniform 4m beam of mass 4kg is supported by a cord attached at the 1m position and at an angle 40°as shown in the diagram.



What force does the hinge exert on the beam? (don't forget about direction) **(3 marks)**

ANSWER: (3 marks)

First, find the tension in the cable.

$$\tau_{cw} = \tau_{ccw}$$

$$\frac{l}{2} \cdot F_{beam} = d \cdot T_{\perp}$$

$$\frac{l}{2} \cdot m_{beam} \cdot g = d \cdot T \cdot \cos(\theta)$$

$$T = \frac{\frac{l}{2} \cdot m_{beam} \cdot g}{d \cdot \cos(\theta)} = \frac{\frac{4m}{2} \cdot 4kg \cdot 9.8m/s^2}{1m \cdot \cos(40°)} = 102.34N$$

Second, find the horizontal force the cord tension exerts on the hinge.

$$\Sigma F_x = 0$$

$$-T_x + F_{x_{hinge}} = 0$$

$$F_{x_{hinge}} = T_x \qquad T_x = T \cdot \sin(\theta)$$

$$F_{x_{hinge}} = T \cdot \sin(\theta) = 102.34N \cdot \sin(40°) = 65.79N$$

Third, find the vertical force the beam exerts on the hinge.

$\Sigma F_y = 0$

$+F_{y_{hinge}} + T_y - F_{beam} = 0$

$F_{y_{hinge}} = F_{beam} - T_y \qquad T_y = T \cdot \cos(\theta)$

$F_{y_{hinge}} = m_{beam} \cdot g - T \cdot \cos(\theta) = 4kg \cdot 9.8m/s^2 - 102.34N \cdot \cos(40°) = -39.2N$

OR we could use $\tau_{cw} = \tau_{ccw}$ to find the vertical force of the hinge but the pivot point is where to cord attaches to the beam.

$\tau_{cw} = \tau_{ccw}$

$d_{wall} \cdot F_{y_{hinge}} = d_{beam} \cdot F_{beam}$

$$F_{y_{hinge}} = \frac{d_{beam} \cdot F_{beam}}{d_{\text{cord}}} = \frac{\left(d_{\text{cord}} - \frac{l}{2}\right) \cdot m_{beam} \cdot g}{d_{\text{cord}}} = \frac{\left(1m - \frac{4m}{2}\right) \cdot 4kg \cdot 9.8m/s^2}{1m} = -39.2N$$

Finally, find the resultant force from the F_x and F_y

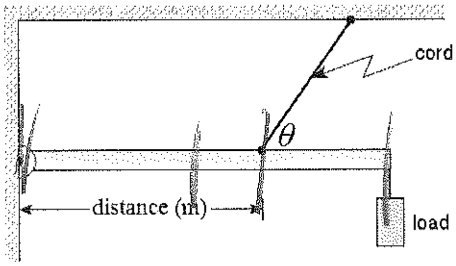
$$\left(F_{hinge}\right)^2 = \left(F_{x_{hinge}}\right)^2 + \left(F_{y_{hinge}}\right)^2$$

$$F_{hinge} = \sqrt{\left(F_{x_{hinge}}\right)^2 + \left(F_{y_{hinge}}\right)^2} = \sqrt{(65.79N)^2 + (-39.2N)^2} = \underline{\underline{76.58N}}$$

$$\tan^{-1}\left(\frac{F_{y_{hinge}}}{F_{x_{hinge}}}\right) = \tan^{-1}\left(\frac{39.2N}{65.79N}\right) = \underline{\underline{30.79°}}$$

The hinge exerts a force of <u>76.58N @ 30.79°</u> *below* the horizontal and to the right.

11. The horizontal uniform beam shown below is 7m long and has a mass of 5kg. At the end of the beam, a load that is 58kg is being held. A cord, at an angle of 69°, is holding the beam. The cord is attached at a distance of 6m from the wall.



What force does the hinge exert on the beam? (don't forget about direction) **(3 marks)**

ANSWER: (3 marks)

First, find the tension in the cable.

$$\tau_{cw} = \tau_{ccw}$$

$$\frac{l}{2} \cdot F_{beam} + l \cdot F_{load} = d \cdot T_{\perp}$$

$$\frac{l}{2} \cdot m_{beam} \cdot g + l \cdot m_{load} \cdot g = d \cdot T \cdot \sin(\theta)$$

$$T = \frac{\frac{l}{2} \cdot m_{beam} \cdot g + l \cdot m_{load} \cdot g}{d \cdot \sin(\theta)} = \frac{\frac{7m}{2} \cdot 5kg \cdot 9.8m/s^2 + 7m \cdot 58kg \cdot 9.8m/s^2}{6m \cdot \sin(69°)} = 740.93N$$

Second, find the horizontal force the cord tension exerts on the hinge.

$$\Sigma F_x = 0$$

$$T_x - F_{x_{hinge}} = 0$$

$$F_{x_{hinge}} = T_x \qquad T_x = T \cdot \cos(\theta)$$

$$F_{x_{hinge}} = T \cdot \cos(\theta) = 740.93N \cdot \cos(69°) = 265.53N \quad \text{(it is pulling to the left)}$$

Third, find the vertical force the beam exerts on the hinge.

$$\sum F_y = 0$$

$$+F_{y_{hinge}} + T_y - F_{beam} - F_{load} = 0$$

$$F_{y_{hinge}} = F_{beam} + F_{load} - T_y \qquad T_y = T \cdot \sin(\theta)$$

$$F_{y_{hinge}} = m_{beam} \cdot g + m_{load} \cdot g - T \cdot \sin(\theta) = 5kg \cdot 9.8m/s^2 + 58kg \cdot 9.8m/s^2 - 740.93N \cdot \sin(69°) = -74.32N$$

OR we could use $\tau_{cw} = \tau_{ccw}$ to find the vertical force of the hinge but the pivot point is where to cord attaches to the beam.

$$\tau_{cw} = \tau_{ccw}$$

$$d_{wall} \cdot F_{y_{hinge}} + d_{load} \cdot F_{load} = d_{beam} \cdot F_{beam}$$

$$F_{y_{hinge}} = \frac{d_{beam} \cdot F_{beam} - d_{load} \cdot F_{load}}{d_{cord}} = \frac{\left(d_{cord} - \frac{l}{2}\right) \cdot m_{beam} \cdot g - \left(l - d_{cord}\right) \cdot m_{load} \cdot g}{d_{cord}}$$

$$F_{y_{hinge}} = \frac{\left(6m - \frac{7m}{2}\right) \cdot 5kg \cdot 9.8m/s^2 - (7m - 6m) \cdot 58 \cdot 9.8m/s^2}{6m} = -74.32N$$

Finally, find the resultant force from the F_x and F_y

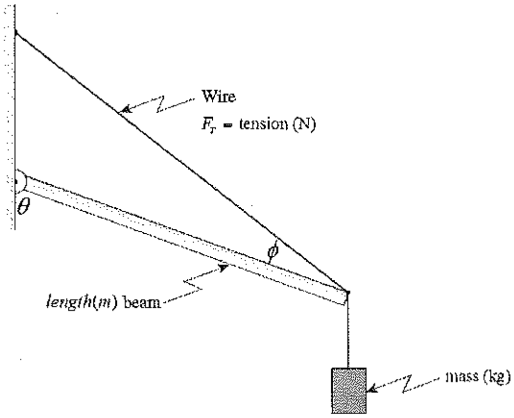
$$\left(F_{hinge}\right)^2 = \left(F_{x_{hinge}}\right)^2 + \left(F_{y_{hinge}}\right)^2$$

$$F_{hinge} = \sqrt{\left(F_{x_{hinge}}\right)^2 + \left(F_{y_{hinge}}\right)^2} = \sqrt{(265.53N)^2 + (-74.32N)^2} = \underline{\underline{275.73N}}$$

$$\tan^{-1}\left(\frac{F_{y_{hinge}}}{F_{x_{hinge}}}\right) = \underline{\underline{15.64°}}$$

The hinge exerts a force of 275.73N @ 15.64° *below* the horizontal and to the left.

12. A 3.9m long uniform beam that has a mass of 33kg, supports a 145kg load. The beam is suspended by a wire connected as shown. Angle θ=61° and angle $\phi = 69°$



What is the tension in the wire? **(3 marks)**

ANSWER: (3 marks)

$$\tau_{cw} = \tau_{ccw}$$

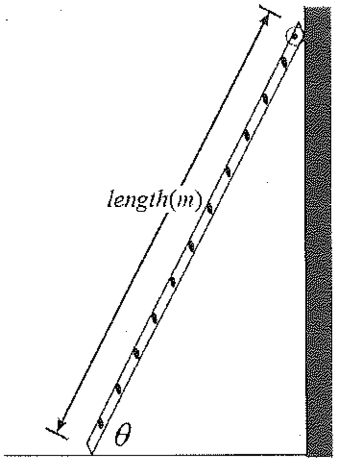
$$\frac{l}{2} \cdot F_{\perp_{beam}} + l \cdot F_{\perp_{load}} = l \cdot T_{\perp}$$

$$\frac{l}{2} \cdot m_{beam} \cdot g \cdot \sin(\theta) + l \cdot m_{load} \cdot g \cdot \sin(\theta) = l \cdot T \cdot \sin(\phi)$$

$$T = \frac{\frac{1}{2} \cdot m_{beam} \cdot g \cdot \sin(\theta) + m_{load} \cdot g \cdot \sin(\theta)}{\sin(\phi)} = \frac{\frac{1}{2} \cdot 33kg \cdot 9.8m/s^2 \cdot \sin(61°) + 145kg \cdot 9.8m/s^2 \cdot \sin(61°)}{\sin(69°)}$$

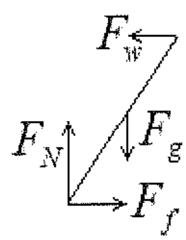
$$T = \underline{\underline{1{,}482.74N}}$$

13. A uniform 5.6m long ladder of mass 21kg leans at an angle of 48°against a **frictionless** vertical wall as shown in the diagram below.



a) Draw and label a free body diagram showing the forces acting on the ladder. **(1 mark)**
ANSWER: (3 marks)

ANSWER:



b) What coefficient of friction is needed at the base of the ladder to keep it from sliding? **(3 marks)**

$F_f = \mu \cdot F_N = \mu \cdot m \cdot g$

$F_f = F_{wall}$ $\quad \sin(\theta) = \frac{F_\perp}{F_{wall}}$ $\quad F_\perp = F_{wall} \cdot \sin(\theta) = F_f \cdot \sin(\theta) = \mu \cdot m \cdot g \cdot \sin(\theta)$

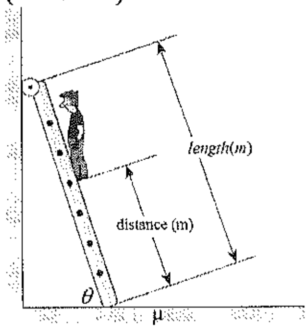
$\tau_{cw} = \tau_{ccw}$

$\frac{l}{2} \cdot F_g \cdot \cos(\theta) = l \cdot F_\perp$

$\frac{l}{2} \cdot m \cdot g \cdot \cos(\theta) = l \cdot \mu \cdot m \cdot g \cdot \sin(\theta)$

$\mu = \frac{\cos(\theta)}{2 \cdot \sin(\theta)} = \frac{1}{2 \cdot \tan(\theta)} = \frac{1}{2 \cdot \tan(48°)} = \underline{\underline{0.45}}$

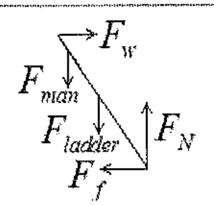
14. A 82kg man is 0.8m up a 4.3m, 18kg ladder leaning against a smooth wall at an angle of 70° as shown below. **(4 marks)**



ANSWER: (4 marks)

a) Draw and label a free body diagram showing the forces acting on the ladder. **(1 mark)**

ANSWER:



b) What is the minimum coefficient of friction required to keep the ladder from sliding? **(3 marks)**

$$\tau_{cw} = \tau_{ccw}$$

$$\frac{l}{2} \cdot F_{g_{ladder}} \cdot \cos(\theta) + d \cdot F_{g_{man}} \cdot \cos(\theta) = l \cdot F_{wall} \cdot \sin(\theta)$$

$$F_{wall} = \frac{\frac{l}{2} \cdot m_{ladder} \cdot g \cdot \cos(\theta) + d \cdot m_{man} \cdot g \cdot \cos(\theta)}{l \cdot \sin(\theta)}$$

$$F_{wall} = \frac{\frac{4.3m}{2} \cdot 18kg \cdot 9.8m/s^2 \cdot \cos(70°) + 0.8m \cdot 82kg \cdot 9.8m/s^2 \cdot \cos(70°)}{4.3m \cdot \sin(70°)} = 86.52N$$

$$\Sigma F_y = 0$$

$$+F_N - F_{g_{ladder}} - F_{g_{man}} = 0$$

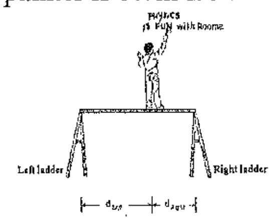
$$F_N = F_{g_{ladder}} - F_{g_{man}} = \left(m_{ladder} + m_{man}\right) \cdot g$$

$$F_f = \mu \cdot F_N = \mu \cdot \left(m_{ladder} + m_{man}\right) \cdot g$$

$$\mu \cdot F_N = F_{wall}$$

$$\mu = \frac{F_{wall}}{F_N} = \frac{F_{wall}}{\left(m_{ladder} + m_{man}\right) \cdot g} = \frac{86.52N}{\left(18kg + 82kg\right) \cdot 9.8m/s^2} = \underline{0.09}$$

15. A 79kg painter stands on a uniform 8m board of mass 12kg supported horizontally by two ladders. The painter is 5.7m from the left ladder and 2.3m from the right ladder.



a) Find the force exerted by the right ladder. **(2 marks)**

a) Force of Right Ladder:

$$\tau_{cw} = \tau_{ccw}$$

$$d_{Left} F_{g_{painter}} + r_b F_{g_{board}} = r_l F_R$$

$$5.7m \cdot 79kg \cdot 9.8m/s^2 + \frac{8m}{2} \cdot 12kg \cdot 9.8m/s^2 = 8m \cdot F_R$$

$$F_R = \frac{5.7m \cdot 79kg \cdot 9.8m/s^2 + \frac{8m}{2} \cdot 12kg \cdot 9.8m/s^2}{8m} = \underline{\underline{610.42N}}$$

b) Force of Left Ladder:

$$\tau_{cw} = \tau_{ccw}$$

$$d_{Right} F_{g_{painter}} + r_b F_{g_{board}} = r_l F_L$$

$$2.3m \cdot 79kg \cdot 9.8m/s^2 + \frac{8m}{2} \cdot 12kg \cdot 9.8m/s^2 = 8m \cdot F_L$$

$$F_L = \frac{2.3m \cdot 79kg \cdot 9.8m/s^2 + \frac{8m}{2} \cdot 12kg \cdot 9.8m/s^2}{8m} = \underline{\underline{281.38N}}$$

OR we could use $\sum F_y = 0$

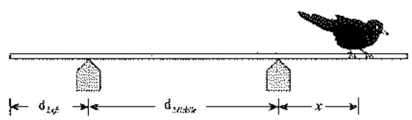
$$F_L + F_R - F_{g_{painter}} - F_{g_{board}} = 0$$

$$F_L = F_{g_{painter}} + F_{g_{board}} - F_R$$

$$F_L = \cdot 79kg \cdot 9.8m/s^2 + \cdot 12kg \cdot 9.8m/s^2 - 610.42N = \underline{\underline{281.38N}}$$

16.

A 2.2 kg board of length 5 m initially rests on two supports as shown. The distance from left support to end is 0.4m and the distance between the two supports is 2.4m.



a) What maximum distance, x, from the right-hand support can a 1.8 kg bird walk before the board begins to leave the left-hand support? **(2 marks)**

a) Distance of bird:

$$\sum\tau_{cw} = \sum\tau_{ccw}$$

$$r \cdot F_{g_{board}} = x \cdot F_{g_{bird}}$$

$$\left(d_{left} + d_{middle} - d_{\frac{board}{2}}\right) \cdot m_{board} \cdot g = x \cdot m_{bird} \cdot g$$

$$x = \frac{\left(d_{left} + d_{middle} - d_{\frac{board}{2}}\right) \cdot m_{board}}{m_{bird}} = \frac{\left(0.4m + 2.4m - \frac{5m}{2}\right) \cdot 2.2kg}{1.8kg} = \underline{\underline{0.67m}}$$

b) Force exerted by right support:

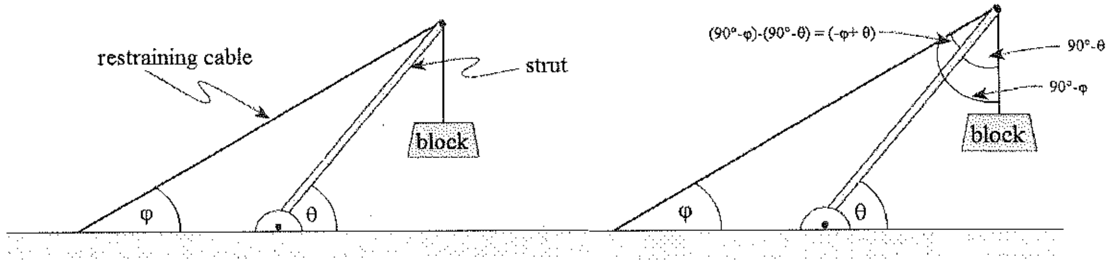
$$\sum F_y = 0$$

$$+F_R - F_{g_{board}} - F_{g_{bird}} = 0$$

$$F_R = F_{g_{board}} + F_{g_{bird}} = m_{board} \cdot g + m_{bird} \cdot g = 2.2kg \cdot 9.8m/s^2 + 1.8kg \cdot 9.8m/s^2 = \underline{\underline{39.2N}}$$

17. 1

18. The crane shown in the diagram below is made up of a strut and a restraining cable. The strut is uniform in cross section with a length of 17.7 m and a mass of 85 kg. What is the tension in the restraining cable while the crane is supporting a 175 kg block?
The angle of the strut to the ground is ▯ =69▯ and the angle of the restraining cable to the ground is ▯=47▯▯ **(3 marks)**



ANSWER: (3 marks)

$$\tau_{cw} = \tau_{ccw}$$

$$\frac{l}{2} \cdot F_{\perp_{beam}} + l \cdot F_{\perp_{load}} = l \cdot T_{\perp}$$

$$\frac{l}{2} \cdot m_{beam} \cdot g \cdot \cos(\theta) + l \cdot m_{load} \cdot g \cdot \cos(\theta) = l \cdot T \cdot \sin((90-\phi)-(90-\theta))$$

$$T = \frac{\frac{1}{2} \cdot m_{beam} \cdot g \cdot \cos(\theta) + m_{load} \cdot g \cdot \cos(\theta)}{\sin(-\phi+\theta)} = \frac{\frac{1}{2} \cdot 85kg \cdot 9.8m/s^2 \cdot \cos(69°) + 175kg \cdot 9.8m/s^2 \cdot \cos(69°)}{\sin(-47° + 69°)}$$

$$T = \underline{\underline{2{,}039.1N}}$$