Physics 12 - Momentum
Answer Section

SHORT ANSWER

1. Which set of conditions is true in all inelastic collisions?

	MOMENTUM	KINETIC ENERGY	TOTAL ENERGY
B.	Conserved	Not conserved	Conserved

ANSWER: B

2. In order to stop two sliding objects, the greater impulse **must** be given to the one having the greater **(1 mark)**
ANSWER:

D. momentum.

3. A puck sliding on a frictionless table undergoes a change in momentum due to a constant force. Which of the following expressions could be used to determine the change in momentum? **(1 mark)**
ANSWER:

B. $F \times \Delta t$

4. A 1.5 kg puck travelling due East at 3.6 m/s collides with a 2.5 kg puck travelling due South at 1.7 m/s. They stick together on impact.
What is the resultant velocity of the combined pucks (and direction)? **(3 marks)**
ANSWER: (3 marks)

$$p_x = mv = 1.5kg \cdot 3.6m/s = 5.4kg \cdot m/s$$

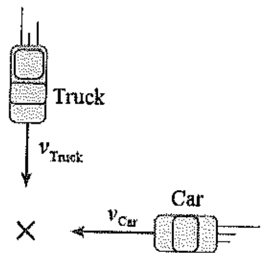
$$p_y = mv = 2.5kg \cdot 1.7m/s = 4.25kg \cdot m/s$$

$$p = \sqrt{(p_x)^2 + (p_y)^2} = \sqrt{(5.4kg \cdot m/s)^2 + (4.25kg \cdot m/s)^2} = 6.87kg \cdot m/s$$

$$p = mv \qquad v = \frac{p}{m_1 + m_2} = \frac{6.87kg \cdot m/s}{(1.5kg + 2.5kg)} = \underline{1.72m/s}$$

$$\tan^{-1}\left(\frac{p_y}{p_x}\right) = \underline{\underline{38.2^\circ}} \text{ (S of E)} \quad \text{OR} \quad \tan^{-1}\left(\frac{p_x}{p_y}\right) = \underline{\underline{51.8^\circ \text{(E of S)}}}$$

5. A 4,450 kg truck travelling due South at 8.8 m/s collides with a 1,950 kg Car travelling due West at 20.8 m/s. The two vehicles stick together after they collide.



a) What is the resultant velocity of the combined vehicles (and direction)? **(2 marks)**
ANSWER: (2 marks)

$$p_x = mv = 1{,}950kg \cdot 20.8m/s = 40{,}560kg \cdot m/s$$

$$p_y = mv = 4{,}450kg \cdot 8.8m/s = 39{,}160kg \cdot m/s$$

$$p = \sqrt{\left(p_x\right)^2 + \left(p_y\right)^2} = \sqrt{\left(40{,}560kg \cdot m/s\right)^2 + \left(39{,}160kg \cdot m/s\right)^2} = 56{,}379.24kg \cdot m/s$$

$$p = mv \qquad v = \frac{p}{m_1 + m_2} = \frac{56{,}379.24kg \cdot m/s}{\left(1{,}950kg + 4{,}450kg\right)} = \underline{\underline{8.81m/s}}$$

$$\tan^{-1}\left(\frac{p_y}{p_x}\right) = \underline{\underline{43.99^\circ}} \text{ (S of W)} \quad \text{OR} \quad \tan^{-1}\left(\frac{p_x}{p_y}\right) = \underline{\underline{46.01^\circ \text{(W of S)}}}$$

b) How much energy was converted to heat during the collision **(2 marks)**

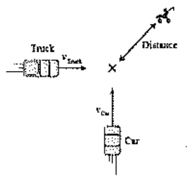
$$E_{K_{Truck}} + E_{K_{Car}} = E_{K_{Both}} + E_H \qquad E_H = E_{K_{Truck}} + E_{K_{Car}} - E_{K_{Both}}$$

$$E_H = \frac{1}{2} m_{Truck}\left(v_{Truck}\right)^2 + \frac{1}{2} m_{Car}\left(v_{Car}\right)^2 - \frac{1}{2} m_{Both}\left(v_{Both}\right)^2$$

$$E_H = \frac{1}{2} \cdot 4{,}450kg \cdot \left(8.8\tfrac{m}{s}\right)^2 + \frac{1}{2} \cdot 1{,}950kg \cdot \left(20.8\tfrac{m}{s}\right)^2 - \frac{1}{2} \cdot \left(4{,}450kg + 1{,}950kg\right) \cdot \left(8.81\tfrac{m}{s}\right)^2$$

$$E_H = 172{,}304J + 421{,}824J - 248{,}329.6J = \underline{\underline{345{,}798.38J}}$$

6. A 2,400 kg truck travelling due East at 10.7 m/s collides with a 1,050 kg Car travelling due North at 12.8 m/s. The two vehicles stick together after they collide. If there is a baby in a stroller23 m from the impact site, does it get hit if there is 9200 N of road friction?



a) What is the resultant velocity of the combined vehicles (and direction)? **(2 marks)**

$p_x = mv = 2{,}400kg \cdot 12.8m/s = 30{,}720kg \cdot m/s$

$p_y = mv = 1{,}050kg \cdot 10.7m/s = 11{,}235kg \cdot m/s$

$$p = \sqrt{\left(p_x\right)^2 + \left(p_y\right)^2} = \sqrt{\left(30{,}720kg \cdot m/s\right)^2 + \left(11{,}235kg \cdot m/s\right)^2} = 32{,}709.99kg \cdot m/s$$

$$p = mv \qquad v = \frac{p}{m_1 + m_2} = \frac{32{,}709.99kg \cdot m/s}{\left(2{,}400kg + 1{,}050kg\right)} = \underline{9.48m/s}$$

$$\tan^{-1}\left(\frac{p_y}{p_x}\right) = \underline{20.09^\circ} \text{ (N of E)} \quad \text{OR} \quad \tan^{-1}\left(\frac{p_x}{p_y}\right) = \underline{69.91^\circ \text{(E of N)}}$$

X $V_F = 8.4$ m/s @ 28° N of E

b) Do the vehicles hit the baby stroller **(2 marks)**

ANSWER: (2 marks)

$W = Fd = \Delta E = E_{Kf} - E_{Ki}$

$$d = \frac{E_{Kf} - E_{Ki}}{F_f} = \frac{0 - \frac{1}{2} m_{Both}\left(v_{Both}\right)^2}{-F_f} = \frac{0 - \frac{1}{2} \cdot \left(1{,}050kg + 2{,}400kg\right) \cdot \left(9.48\frac{m}{s}\right)^2}{-9200N} = 16.85m$$

If the distance travelled is greater than the distance the stroller is away from the crash site, then the stroller is hit.

distance travelled = ~~16.85m~~ 13 m   distance of stroller = ~~23m~~ 18 m 23 m

The vehicles *miss* the stroller.

X

7. A 4.6 kg block sliding at 4.8 m/s across a horizontal frictionless surface collides head on with a stationary 4.8 kg block. The 4.6 kg block rebounds at 2.3 m/s.
How much kinetic energy is lost during this collision? **(3 marks)**
ANSWER: (3 marks)

$$p_1 + p_2 = p_1' + p_2'$$

$$m_1 v_1 = m_1 v_1' + m_2 v_2'$$

$$v_2' = \frac{m_1 v_1 - m_1 v_1'}{m_2} = \frac{4.6kg \cdot 4.8m/s - 4.6kg \cdot 2.3m/s}{4.8kg} = 2.4m/s$$

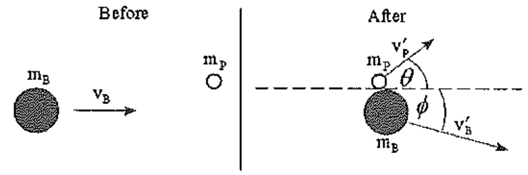
$$E_{K_1} + E_{K_2} = E_{K_1}' + E_{K_2}' + E_H$$

$$E_H = E_{K_1} - E_{K_1}' - E_{K_2}' = \frac{1}{2}m_1\left(v_1\right)^2 - \frac{1}{2}m_1\left(v_1'\right)^2 - \frac{1}{2}m_2\left(v_2'\right)^2$$

$$E_H = \frac{1}{2}4.6kg(4.8m/s)^2 - \frac{1}{2}4.6kg(2.3m/s)^2 - \frac{1}{2}4.8kg(2.4m/s)^2$$

$$E_H = 52.99J - 12.17J - 13.78J = \underline{\underline{27.05J}}$$

8. A 5.4 kg bowling ball travelling 4.5 m/s collides with a stationary 0.7 kg bowling pin. After the collision, the pin moves at a speed of 4.8 m/s at an = 52 (N of E)shown in the diagram.



What is the velocity (magnitude and direction) of the bowling ball after the collision?
ANSWER: (3 marks)
X-Component:

$$p_{x_{Ball}} + p_{x_{Pin}} = p'_{x_{Ball}} + p'_{x_{Pin}}$$

$$p'_{x_{Ball}} = p_{x_{Ball}} - p'_{x_{Pin}} = m_B v_B - m_P v'_P \cos(52°) = 5.4kg \cdot 4.5m/s - 0.7kg \cdot 4.8m/s\cos(52°) = 22.23kg \cdot m/s$$

Y-Component:

$$p_{y_{Ball}} + p_{y_{Pin}} = p'_{y_{Ball}} + p'_{y_{Pin}}$$

$$p'_{y_{Ball}} = -p'_{y_{Pin}} = -m_P v'_P \sin(52°) = 0.7kg \cdot 4.8m/s \cdot \sin(52°) = -2.65kg \cdot m/s$$

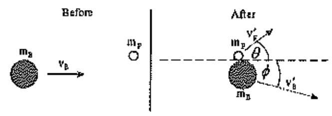
$$p = \sqrt{\left(p'_{x_{Ball}}\right)^2 + \left(p'_{y_{Ball}}\right)^2} = \sqrt{\left(22.23kg \cdot m/s\right)^2 + \left(-2.65kg \cdot m/s\right)^2} = 22.39kg \cdot m/s$$

$$p = m_B v_B \qquad v_B = \frac{p}{m_B} = \frac{22.39kg \cdot m/s}{5.4kg} = \underline{\underline{4.15m/s}}$$

$$\tan^{-1}\left(\frac{p_y}{p_x}\right) = \underline{\underline{6.79°}} \text{ (S of E)} \quad \text{OR} \quad \tan^{-1}\left(\frac{p_x}{p_y}\right) = \underline{\underline{83.21°\text{(E of S)}}}$$

9. Mr. Roome is all alone in the physics lab on a Friday night. He sets up the air table and does some conservation of momentum labs to pass the time before he sleeps under his desk, waiting for Monday to arrive. Just before Mr. Roome turns into the living dead at midnight, he scrawls on a piece of lab paper... $m_B = 5.9kg$, $v_B = 3.2m/s$, $\phi = 34°$, $v'_B = 2.2m/s$, $v'_P = 4.1m/s$. He also writes that the Accepted value is 2.58 kg.

$$\%error = \frac{|Accepted - Measured|}{Accepted} * 100\%$$



a) Using his results from the diagram above, determine the mass of m_P **(3 marks)**

ANSWER: (3 marks)

X-Component:

$$p_{x_{Ball}} + p_{x_{Pin}} = p'_{x_{Ball}} + p'_{x_{Pin}}$$

$$p'_{x_{Pin}} = p_{x_{Ball}} - p'_{x_{Ball}} = m_B v_B - m_B v'_B \cos(34°) = 5.9kg \cdot 3.2m/s - 5.9kg \cdot 2.2m/s \cdot \cos(34°) = 8.12kg \cdot m/s$$

Y-Component:

$$p_{y_{Ball}} + p_{y_{Pin}} = p'_{y_{Ball}} + p'_{y_{Pin}}$$

$$p'_{y_{Pin}} = -p'_{y_{Ball}} = -m_B v'_B \sin(34°) = 5.9kg \cdot 2.2m/s \cdot \sin(34°) = -7.26kg \cdot m/s$$

$$p = \sqrt{\left(p'_{x_{Ball}}\right)^2 + \left(p'_{y_{Ball}}\right)^2} = \sqrt{\left(8.12kg \cdot m/s\right)^2 + \left(-7.26kg \cdot m/s\right)^2} = 10.89kg \cdot m/s$$

$$p = m_B v_B \qquad m_P = \frac{p}{v_P} = \frac{10.89kg \cdot m/s}{4.1m/s} = \underline{\underline{2.66kg}}$$

$$\tan^{-1}\left(\frac{p_y}{p_x}\right) = \underline{\underline{41.8°}} \text{ (N of E)} \quad \text{OR} \quad \tan^{-1}\left(\frac{p_x}{p_y}\right) = \underline{\underline{48.2° \text{(E of N)}}}$$

b) Now find the % error of his work. **(1 mark)**

$$\%\ error = \left|\frac{Accepted - Measured}{Accepted}\right| \times 100\% = \left|\frac{2.58kg - 2.66kg}{2.58kg}\right| \times 100\% = \underline{\underline{3.11\%}}$$

10. A 0.1 kg ball moving at 16 m/s is struck by a bat. The bat reverses the ball's direction and gives it a speed of 32 m/s.

What average force does the bat apply to the ball if they are in contact for 5.3×10^{-3} s? **(3 marks)**

ANSWER: (3 marks)

$$F\Delta t = \Delta p = m\Delta v = m\left(v_f - v_i\right)$$

$$F = \frac{m\left(v_f - v_i\right)}{\Delta t} = \frac{0.1kg(-32m/s - 16m/s)}{5.3 \times 10^{-3}s} = \underline{\underline{-905.66N}} \text{ (same direction as final velocity)}$$

11. A 4,050 kg space vehicle consists of a 2,500 kg main capsule and a 1,550 kg probe. The space vehicle is travelling at 80 m/s when an explosion occurs between the capsule and the probe. As a result, the probe moves forward at 104 m/s, as shown in the diagram below.

a) What is the speed of the main capsule **after** the explosion? **(3 marks)**
ANSWER: (3 marks)
$p_i = p_f$

$$\left(m_{capsule} + m_{probe}\right)v = m_{capsule}v_{capsule} + m_{probe}v_{probe} \qquad v_{capsule} = \frac{\left(m_{capsule} + m_{probe}\right)v - m_{probe}v_{probe}}{m_{capsule}}$$

$$v_{capsule} = \frac{\left(2{,}500kg + 1{,}550kg\right)\cdot 80m/s - 1{,}550kg\cdot 104m/s}{2{,}500kg} = \underline{\underline{65.12m/s}}$$

b) What is the magnitude of the impulse given to the probe? **(1 mark)**
$impulse = \Delta p = p_f - p_i = m(v_f - v_i) = 1{,}550kg\cdot(104m/s - 80m/s) = \underline{\underline{37{,}200kgm/s}}$

12. A 0.2 kg soccer ball is kicked with an initial velocity of 9.3 m/s at an angle of 75° above the horizontal (air resistance is negligible).
What is the soccer ball's change in momentum during the time in the air? **(3 marks)**
ANSWER: (3 marks)
$v_f = 0\,m/s\,(\max height)$

First determine how long the ball is in the air.

$v_i = 9.3\,m/s$

$v_{iy} = 9.3\cdot\sin(75°) = 8.98\,m/s$

$v_{fy} = -8.98m/s$

$a = -9.8\,m/s^2$

$t = ?\,s$

$v_f = v_i + at$

$$t = \frac{v_f - v_i}{a} = \frac{-8.98m/s - 8.98m/s}{-9.8m/s^2} = 1.83s$$

Now determine the impulse on the ball from gravity.

$$F\Delta t = \Delta p \qquad \Delta p = F\Delta t = mg\Delta t = 0.2kg\cdot 9.8\frac{N}{kg}\cdot 1.83s = \underline{\underline{3.59kg\cdot m/s}}\ \text{(Down)}$$

13. A 41 kg object is moving due north at 35 m/s. The impulse of 175 N·s is applied at 41° N of E to this object.
What is the final velocity of this object and direction? **(3marks)**
ANSWER: (3 marks)
x – component: $impulse = \Delta p = p_{f_x} - p_{i_x}$ (because it is moving only North, there was no p_x initially)

$impulse \cdot \cos\theta = p_{f_x} \quad p_{f_x} = impulse \cdot \cos\theta = 175N \cdot s \cdot \cos(41°) = 132.07kgm/s$

y – component: $impulse = \Delta p = p_{f_y} - p_{i_y} \quad impulse \cdot \sin\theta = p_{f_y} - p_{i_y}$

$p_{f_y} = impulse \cdot \sin\theta + mv = 175N \cdot s \cdot \sin(41°) + 41kg \cdot 35m/s = 1{,}549.81kgm/s$

$$p_T = \sqrt{\left(p_{f_x}\right)^2 + \left(p_{f_y}\right)^2} = 1{,}555.43m/s \quad \tan^{-1}\left(\frac{p_{f_y}}{p_{f_x}}\right) = \tan^{-1}\left(\frac{1{,}549.81m/s}{132.07m/s}\right) = 85.1° NofE\ OR\ 4.9° EofN$$

$p = mv$

$$v = \frac{p}{m} = \frac{1{,}555.43m/s}{41kg} = \underline{\underline{37.94m/s \quad @ \quad 85.1° NofE \quad OR \quad 4.9° EofN}}$$

14. A 4,600 kg truck travelling at 29 m/s in the direction of 20° N of E collides with a stationary 1,400 kg car. After the collision, the 1,400kg car has a speed of 25 m/s due east.
What is the resulting speed of the truck? **(3marks)**
ANSWER: (3 marks)
x – component:

$p_{1_x} + p_{2_x} = p'_{1_x} + p'_{2_x}$

$p'_{1_x} = p_{1_x} - p'_{2_x} = m_{1_x} v_{1_x} - m_{2_x} v'_{2_x} = 4{,}600kg \cdot 29m/s \cdot \cos(20°) - 1{,}400kg \cdot 25m/s = 90{,}355kgm/s$

y – component:

$p_{1_y} + p_{2_y} = p'_{1_y} + p'_{2_y}$

$p'_{1_y} = p_{1_y} = m_{1_y} v_{1_y} = 4{,}600kg \cdot 29m/s \cdot \sin(20°) = 45{,}625.49kgm/s$

$$p_T = \sqrt{\left(p_{f_x}\right)^2 + \left(p_{f_y}\right)^2} = 101{,}221.1m/s$$

$$\tan^{-1}\left(\frac{p_{f_y}}{p_{f_x}}\right) = \tan^{-1}\left(\frac{45{,}625.49m/s}{90{,}355m/s}\right) = 26.8° NofE\ OR\ 63.2° EofN$$

$p = mv$

$$v = \frac{p}{m} = \frac{101{,}221.1m/s}{4{,}600kg} = \underline{\underline{22m/s \quad @ \quad 26.8° NofE \quad OR \quad 63.2° EofN}}$$

15. Outside the International Space Station, a 96 kg astronaut holding a 8.4 kg object (both initially at rest) throws the object at 18 m/s relative to the space station. A 60 kg astronaut, initially at rest, catches the object.
What is the speed of separation of the two astronauts? **(4 marks)**
ANSWER: (4 marks)
The first event is an explosion (throwing the object)

$$\left(m_1 + m_2\right)v = m_1 v_1' + m_2 v_2'$$

$$v_1' = \frac{m_2 v_2'}{m_1} = \frac{8.4kg \cdot 18m/s}{96kg} = 1.58m/s$$

The second event is a collision (catching the object)

$$m_2 v_2 + m_3 v_3 = \left(m_2 + m_3\right) \cdot v_3'$$

$$v_3' = \frac{m_2 v_2}{\left(m_2 + m_3\right)} = \frac{8.4kg \cdot 18m/s}{\left(8.4kg + 60kg\right)} = 2.21m/s$$

The speed of separation is the velocities of the two astronauts combined

$$v_{separation} = v_1' + v_3' = 1.58m/s + 2.21m/s = \underline{\underline{3.79m/s}}$$

16. A 2.7 kg rocket is moving at 100m/s when the second stage of the rocket, capable of generating a 365 N·s impulse, is ignited.
a) What will the velocity of the rocket be immediately after the rocket motor burns out? **(2 marks)**
ANSWER: (4 marks)
$impulse = \Delta p = m(v_f - v_i)$

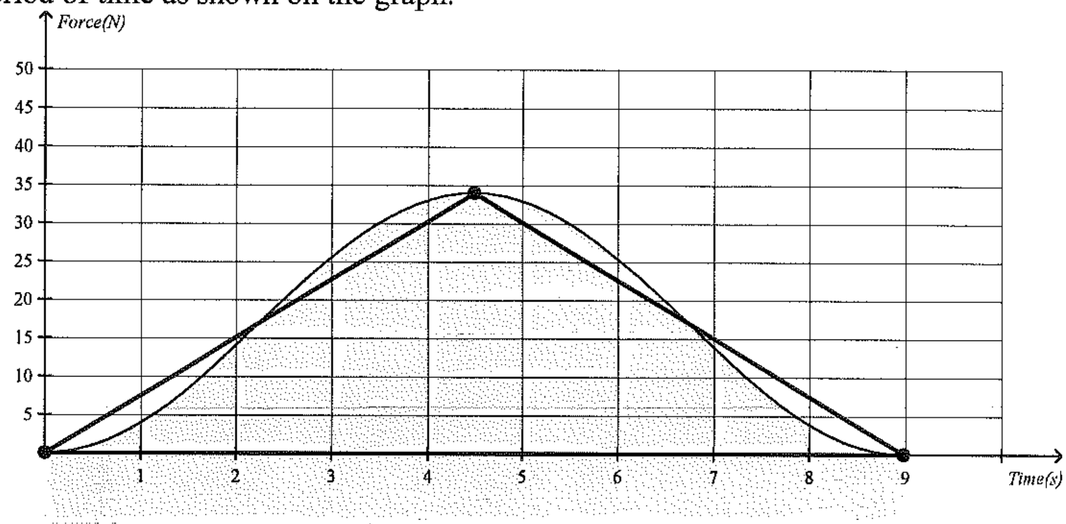
$$v_f = \frac{impulse}{m} + v_i = \frac{365N \cdot s}{2.7kg} + 100m/s = \underline{\underline{235.19m/s}}$$

b) If the rocket can produce 194 N of force during the duration, how far did the rocket travel during the second stage burn? **(2 marks)**
$W = Fd$

$$d = \frac{W}{F} = \frac{\Delta E}{F} = \frac{E_{k_f} - E_{k_i}}{F} = \frac{\frac{1}{2}m\left(v_f\right)^2 - \frac{1}{2}m\left(v_i\right)^2}{F} = \frac{\frac{1}{2} \cdot 2.7kg\left(235.19\frac{m}{s}\right)^2 - \frac{1}{2} \cdot 2.7kg\left(100\frac{m}{s}\right)^2}{194N}$$

$$d = \frac{74,671.3J - 13,500J}{194N} = \frac{61,171.296J}{194N} = \underline{\underline{315.32m}}$$

17. A 9 kg model vehicle travelling at 5 m/s [to the right] experiences a push [to the left] for a certain period of time as shown on the graph.



What is the resulting velocity and indicate the direction of motion [Left or Right]? **(3 marks)**
ANSWER: (3 marks)

$Area = \frac{1}{2} b \cdot h = \frac{1}{2}(8.98)(34) = \underline{152.59 N \cdot s}$

$area\ under\ the\ curve = \underline{129.91 N \cdot s}$

$area = -impulse = \Delta p = m\Delta v = m\left(v_f - v_i\right)$ (because the impulse is to the left, it is a negative value)

$v_f = \frac{-impulse}{m} + v_i = \frac{-129.91 N \cdot s}{9kg} + 5m/s = \underline{\underline{-9.43 m/s\ [left]}}$

OR using the triangle area $v_f = \frac{-152.59 N \cdot s}{9kg} + 5m/s = \underline{\underline{-11.95 m/s\ [left]}}$

18. A 15 kg red ball is moving with a velocity of 1 m/s to the right when it has a perfectly elastic collision with a 13 kg green ball heading left at 10 m/s.
What are the velocities of the two balls after the collision (state direction, left or right)?

(3 marks)

Conservation of Momentum:

$$p_1 + p_2 = p_1' + p_2'$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad \text{(isolate } v_1'\text{)}$$

$$v_1' = \frac{m_1 v_1 + m_2 v_2 - m_2 v_2'}{m_1} = \frac{\left(15kg\right)\cdot\left(1\tfrac{m}{s}\right) + \left(13kg\right)\cdot\left(-10\tfrac{m}{s}\right) - \left(13kg\right)\cdot v_2'}{15kg} \qquad v_1' = -7.67 - 0.87v_2'$$

Conservation of Energy:

$$E_{k_1} + E_{k_2} = E_{k_1}' + E_{k_2}'$$

$$\tfrac{1}{2} m_1 \left(v_1\right)^2 + \tfrac{1}{2} m_2 \left(v_2\right)^2 = \tfrac{1}{2} m_1 \left(v_1'\right)^2 + \tfrac{1}{2} m_2 \left(v_2'\right)^2 \quad \text{(substitute } v_1' = -7.67 - 0.87v_2'\text{)}$$

$$\tfrac{1}{2}\left(15kg\right)\left(1\tfrac{m}{s}\right)^2 + \tfrac{1}{2}\left(13kg\right)\left(-10\tfrac{m}{s}\right)^2 = \tfrac{1}{2}\left(15kg\right)\left(-7.67 - 0.87v_2'\right)^2 + \tfrac{1}{2}\left(13kg\right)\left(v_2'\right)^2$$

$$7.5 + 650 = 440.83 - -99.67v_2' + 5.63\left(v_2'\right)^2 + 6.5\left(v_2'\right)^2$$

$$12.13\left(v_2'\right)^2 - -99.67\left(v_2'\right) - 216.67 = 0 \qquad ax^2 + bx + c = 0 \qquad x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$v_2' = 1.79\tfrac{m}{s}$ AND $v_2' = -10\tfrac{m}{s}$ REJECT original velocity

$$v_1' = -7.67 - 0.87(1.79) = -9.21\tfrac{m}{s}$$

Red Ball $= -9.21\tfrac{m}{s}\,[left]$ AND Green Ball $= 1.79\tfrac{m}{s}\,[right]$

19. A 3 kg red ball is moving with a velocity of 3 m/s to the right when it has a perfectly elastic collision with a 4 kg green ball heading left at 4 m/s.
What are the velocities of the two balls after the collision (state direction, left or right)?
HINT: What is the most important thing you learned in grade 4?
ANSWER: Printing neatly **(3 marks)**
Conservation of Momentum:

$$p_1 + p_2 = p_1' + p_2'$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad \text{(isolate } v_1'\text{)}$$

$$v_1' = \frac{m_1 v_1 + m_2 v_2 - m_2 v_2'}{m_1} = \frac{\left(3kg\right)\cdot\left(3\tfrac{m}{s}\right) + \left(4kg\right)\cdot\left(-4\tfrac{m}{s}\right) - \left(4kg\right)\cdot v_2'}{3kg} \qquad v_1' = -2.33 - 1.33v_2'$$

Conservation of Energy:

$$E_{k_1} + E_{k_2} = E_{k_1}' + E_{k_2}'$$

$$\tfrac{1}{2}m_1\left(v_1\right)^2 + \tfrac{1}{2}m_2\left(v_2\right)^2 = \tfrac{1}{2}m_1\left(v_1'\right)^2 + \tfrac{1}{2}m_2\left(v_2'\right)^2 \quad \text{(substitute } v_1' = -2.33 - 1.33v_2'\text{)}$$

$$\tfrac{1}{2}\left(3kg\right)\left(3\tfrac{m}{s}\right)^2 + \tfrac{1}{2}\left(4kg\right)\left(-4\tfrac{m}{s}\right)^2 = \tfrac{1}{2}\left(3kg\right)\left(-2.33 - 1.33v_2'\right)^2 + \tfrac{1}{2}\left(4kg\right)\left(v_2'\right)^2$$

$$13.5 + 32 = 8.17 - -9.33v_2' + 2.67\left(v_2'\right)^2 + 2\left(v_2'\right)^2$$

$$4.67\left(v_2'\right)^2 - -9.33\left(v_2'\right) - 37.33 = 0 \qquad ax^2 + bx + c = 0 \qquad x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$v_2' = 2\tfrac{m}{s}$ AND $v_2' = -4\tfrac{m}{s}$ REJECT original velocity

$$v_1' = -2.33 - 1.33(2) = -5\tfrac{m}{s}$$

Red Ball $= -5\tfrac{m}{s}\left[left\right]$ AND Green Ball $= 2\tfrac{m}{s}\left[right\right]$

20. There are two canoes at rest on the lake. One canoe has two people in it that weighs 1,105 N and the other canoe has three people and it weighs 1,875 N. A person from one canoe pushed the other canoe with a force of 165 N for 1.1 s.
What is the speed of separation of the two canoes? **(3 marks)**

ANSWER: (3 marks)

$$W_1 = m_1 g \quad m_1 = \frac{W_1}{g} = \frac{1,105N}{9.8N/kg} = 112.76kg \qquad W_2 = m_2 g \quad m_2 = \frac{W_2}{g} = \frac{1,875N}{9.8N/kg} = 191.33kg$$

$$F\Delta t = m\Delta v$$

$$v_f - v_i = \frac{F\Delta t}{m_1} = \frac{165N \cdot 1.1s}{112.76} = 1.61m/s$$

$$v_f - v_i = \frac{F\Delta t}{m_2} = \frac{165N \cdot 1.1s}{191.33} = 0.95m/s$$

Finally, add the two velocities together to get the separation velocity.

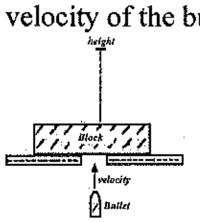
$$v_{separation} = v_1 + v_2 = 1.61m/s + 0.95m/s = \underline{2.56m/s}$$

21. A rocket engine consumes 470 kg of fuel per minute. If the exhaust speed of the ejected fuel is 5.1 km/s, what is the thrust [force] of the rocket? **(3 marks)**
ANSWER: (3 marks)

$$F\Delta t = m\Delta v$$

$$F = \frac{m\Delta v}{\Delta t} = \frac{470kg \cdot (5.1km/s \cdot 1000m/km)}{60s} = \underline{39,950N}$$

22. A 0.80 kg bullet moving at a certain velocity strikes and sticks in the 2.2 kg block initially at rest, as shown below. If the block (with the bullet embedded) rises 19.7 m above it's original position, what was the initial velocity of the bullet? **(3 marks)**



ANSWER: (3 marks)

$$E_{T_{inital}} = E_{T_{final}}$$

$$E_P + E_K = E'_P + E'_K$$

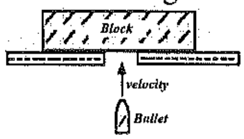
$$\tfrac{1}{2} mv^2 = mgh \qquad v = \sqrt{2gh} = \sqrt{2 \cdot 9.8N/kg \cdot 19.7m} = 19.65m/s$$

$$p_{initial} = p_{final}$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

$$v_1 = \frac{(m_1 + m_2) v'}{m_1} = \frac{(0.80kg + 2.2kg)19.65m/s}{0.80kg} = \underline{73.69m/s}$$

23. A 0.20 kg bullet moving 890 m/s strikes and sticks in the 8.4 kg block initially at rest, as shown below. What maximum height will the block (with the bullet embedded) rise above its initial position? **(3 marks)**



ANSWER: (3 marks)

$p_{initial} = p_{final}$

$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$

$v' = \dfrac{m_1 v_1}{(m_1 + m_2)} = \dfrac{0.20kg \cdot 890 m/s}{(0.20kg + 8.4kg)} = 20.70 m/s$

$E_{T_{initial}} = E_{T_{final}}$

$E_P + E_K = E'_P + E'_K$

$\frac{1}{2} m v^2 = m g h$

$h = \dfrac{v^2}{2g} = \dfrac{(20.70 m/s)^2}{2 \cdot 9.8 N/kg} = \underline{\underline{21.86 m}}$